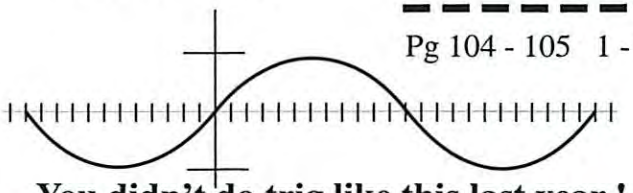
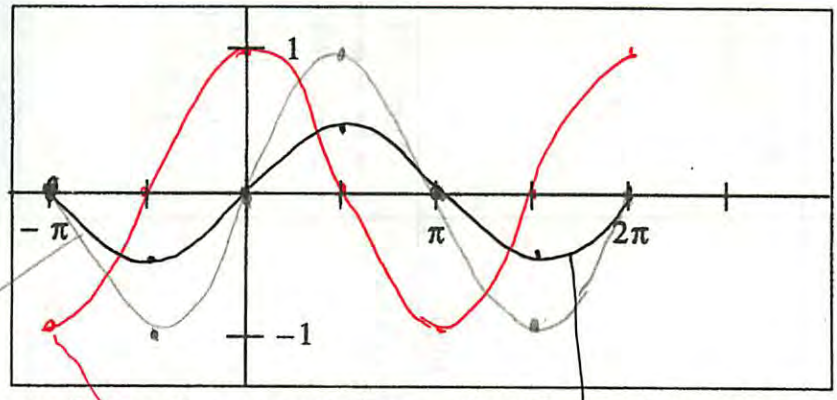
HONORS TRIGONOMETRY CHAPTER 3	**Grade HW6 as such:** **1 - 12 0** **1-12, 1, 2 check** **1-12, 1, 2 , 3 +**	EXAM CHAPTER 2 #1 R/C Pgs 32 - 35 #2 Do Part I on back of sked 10 / 9	#3 Do part II on back of sked **Hand back exams** 10/10	# 4,# 5 Pg 82-83 Change text instructions to read: **graph each and also state Domain & Range** 1 – 16 **Y = A cos x + D** **Graphs of other 4 fctns.** **Show asymptotes** **NO phase shift or period change** *5 page lecture* 10/11
# 6 Same instructions as HW # 4, 5 Pg 92 1 - 12 Pg 88 1 – 6 **Generic graphs MUST be included!!!** **Compute asymptotes, phase shift, period change.** **"Use Asymptotes" lecture** 10/14	Pg 88 7 – 12 Pg 94 6, 5, 4, 3, 11 # 7 R/C Pg 95 - 98 # 8 **Show on calculators. Require students to show 4 coords from both fctns and from the result to show that we are just adding y-values** 10/15	Pg 98 - 100 1 - 10 # 9 **IMPORTANT LECTURE** **Uniform Circular Motion** 10/16	# 10, # 11 Pg 98 - 100 11 - 18 On problem 11, use 2 seconds and – 2 seconds instead of the + 3 and – 3 as listed in the text. 10/17	#12 ,13, 14, 15, 16 Jake's Graphing Adventure **See note on review day** 10/18
Continue on Jakes's Graphing Adventure **Be prepared to turn in tomorrow.** **1st show pblm 15, 16 pg 99.** **Have rqponmlk put on boards.** 10/21	Begin **TEACUPS** # 17, 18, 19, 20, 21 **Collect JGA at beginning of period. Grading Rubric in notebook** 10/22	Continue on TEACUPS **Talk about TC I - III** 10/23	Continue on TEACUPS El Quizzo ??? **Quiz # 1. Sin, Cos graphs any error is an error grade quiz very hard!** **Discuss TC IV - VI** 10/24	Begin: WEEKEND (not so) QUICKIE # 22, 23, 24, 25, 26 **Quiz # 2 All Fctns, phase shift, etc.** 10/25
Turn in TEACUPSMore time on Weekend Quickie Teacup Grading: #17 1 2 3 #18 4 5 Done + #19 6 Trial ✓ #20 7 Pool Part 0 #4 8 El Quizzo ??? 10/28	Last day on WQ  **You didn't do trig like this last year !** 10/29	Turn in the Quickie (end of period) Pg 104 - 105 1 - 6 # 27 10/30	Review Day of Jake's Graph Adv--I do first pblm & students ONLY write the INSTRUCTIONS of what I do NOT the actual work I do. THEN they try to follow thier own directions to do it on another problem. 10/31	**EXAM** **CHAPTER 3** **2002** 11/1

I For each of the following problems, draw and label an axis about 2 inches high and 4 inches long like this:



Plot the coordinates. Draw a smooth curve through the points. No straight lines and no sharp corners are allowed!

1)	2)	3)
$(-\pi, 0)$	$(-\pi, -1)$	$(-\pi, 0)$
$\left(-\frac{\pi}{2}, -1\right)$	$\left(-\frac{\pi}{2}, 0\right)$	$\left(-\frac{\pi}{2}, -\frac{1}{2}\right)$
$(0, 0)$	$(0, 1)$	$(0, 0)$
$\left(\frac{\pi}{2}, 1\right)$	$\left(\frac{\pi}{2}, 0\right)$	$\left(\frac{\pi}{2}, \frac{1}{2}\right)$
$(\pi, 0)$	$(\pi, -1)$	$(\pi, 0)$
$\left(\frac{3\pi}{2}, -1\right)$	$\left(\frac{3\pi}{2}, 0\right)$	$\left(\frac{3\pi}{2}, -\frac{1}{2}\right)$
$(2\pi, 0)$	$(2\pi, 1)$	$(2\pi, 0)$

II A) Draw an axis labeled along the x-axis from $-\pi$ to 3π and along the y-axis from -2 to $+2$. Use the following domain values to create a list of coordinates for each of the functions. Plot the points for all functions on the one graph. Use a different color for each function.

$$x \;\varepsilon\; \left\{-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, 3\pi\right\}$$

$$f(x) = \cos x\,; \quad g(x) = 2\cos x\,; \quad h(x) = \frac{1}{2}\cos x$$

B) Draw an axis labeled along the x-axis from $-\pi$ to 3π and along the y-axis from -2 to $+2$. Use the following domain values to create a list of coordinates for each of the functions. Plot the points for all functions on the one graph. Use a different color for each function. (A calculator may be helpful. Decimal values may be used.

$$x \;\varepsilon\; \left\{-\pi, -\frac{2\pi}{3}, -\frac{\pi}{2}, -\frac{\pi}{3}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{7\pi}{3}, 2\pi, \frac{5\pi}{2}, 3\pi\right\}$$

$$f(x) = \cos x\,; \quad g(x) = \cos 2x\,; \quad h(x) = \cos 3x$$

C) Draw an axis labeled along the x-axis from $-\pi$ to 3π and along the y-axis from -2 to $+2$. Use the following domain values to create a list of coordinates for each of the functions. Plot the points for all functions on the one graph. Use a different color for each function. (A calculator may be helpful. Decimal values may be used.

$$x \;\varepsilon\; \left\{-\pi, -\frac{2\pi}{3}, -\frac{\pi}{2}, -\frac{\pi}{3}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{7\pi}{3}, 2\pi, \frac{5\pi}{2}, 3\pi\right\}$$

$$f(x) = \sin x - 1\,; \quad g(x) = \sin x\,; \quad h(x) = \sin x + 1$$

HONORS TRIGONOMETRY CHAPTER 3	EXAM CHAPTER 2 #1 R/C Pgs 32 - 35 #2 Do Part I on back of sked 10/10	#3 Do part II on back of sked 10/11	# 5 Pg 82-83 Change text instructions to read: **graph each and also state Domain & Range** 1 – 16 10/12	# 6 Same instructions as HW # 5 Pg 92 1 - 12 Pg 88 1 – 6 **Generic graphs MUST be included!!!** 10/13
Pg 88 7 – 12 Pg 94 6, 5, 4, 3, 11 # 7 R/C Pg 95 - 98 # 8 10/16	Pg 98 - 100 1 - 10 # 9 **IMPORTANT LECTURE** 10/17	Pg 98 - 100 11 - 15 # 10 10/18	Pg 98 - 100 16 - 18 # 11 10/19	#12 Jake's Graphing Adventure 10/20
Begin TEACUPS # 13, 14, 15, 16, 17 10/23	Continue on TEACUPS 10/24	Continue on TEACUPS El Quizzo ??? 10/25	Continue on TEACUPS 10/26	Turn in TEACUPS Begin: WEEKEND (not so) QUICKIE # 18, 19, 20, 21, 21 El Quizzo ??? 10/27
Last day on WQ 10/30	Turn in the Quickie Pg 104 - 105 1 - 6 # 23 10/31	Student Holiday 11/1	Review didn't do trig like this last year ! 11/2	EXAM CHAPTER 3 1995 11/3

Honors Trigonometry

El Quizzo # 2

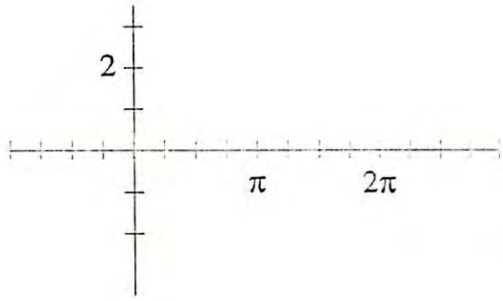
6 +
5 ✓
4 0
3 −

Name _____

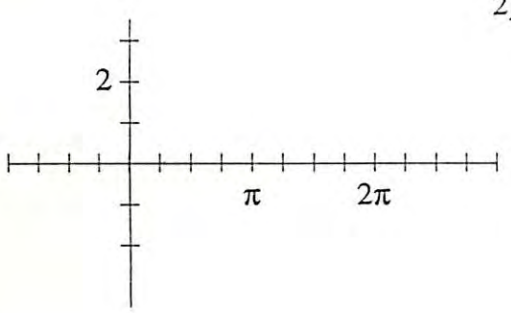
Per _____ Date _____

For each of the following, state Range, Domain, Period, Phase shift (including direction) and amplitude. Graph on the supplied axis.

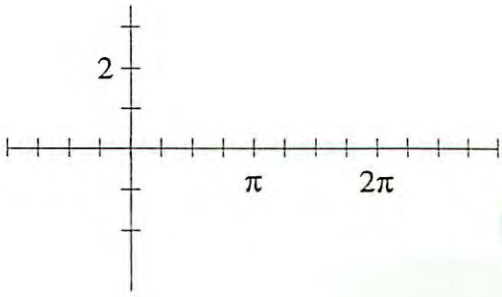
1) $F(x) = \csc\left(\frac{2}{3}x + \frac{\pi}{6}\right) - 1$



2) $F(x) = \cot\left(\frac{2}{3}x + \frac{\pi}{6}\right) + 1$



3) $F(x) = 2\sin\left(\frac{2}{3}x + \frac{\pi}{6}\right)$



$3\cos\frac{\pi}{3}\left(x+\frac{\pi}{3}\right)$

$3\cos\left(\frac{\pi}{3}x+\frac{\pi}{9}\right)$

$\omega(t+b)$

$(\omega t+z)$ z is Head Start

$\frac{2\pi}{\pi}=6$

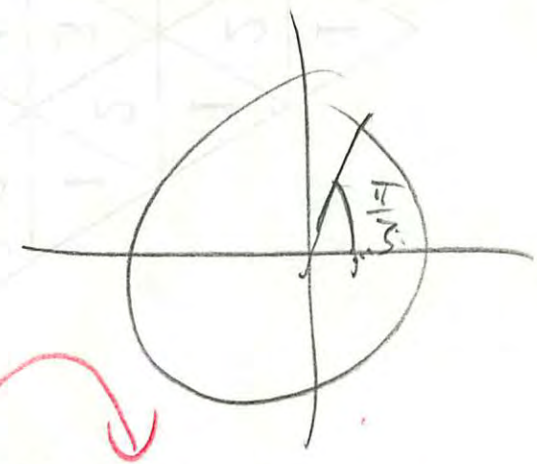
$\omega\left(t+\frac{z}{\omega}\right)$ $\frac{z}{\omega}$ is phase shift.

"Head Start" ≠ Phase Shift

$\cos(\underline{\omega t}+b)$

$\omega(t+b)$

$\underline{\omega b}$



when $t=0$

$\frac{\pi}{3}(\cancel{x}+1)$

P.S. is one but Head start is $\frac{\pi}{3}$.

HONORS TRIG CHAPT 3 **UNIFORM CIRCULAR MOTION**

Cos

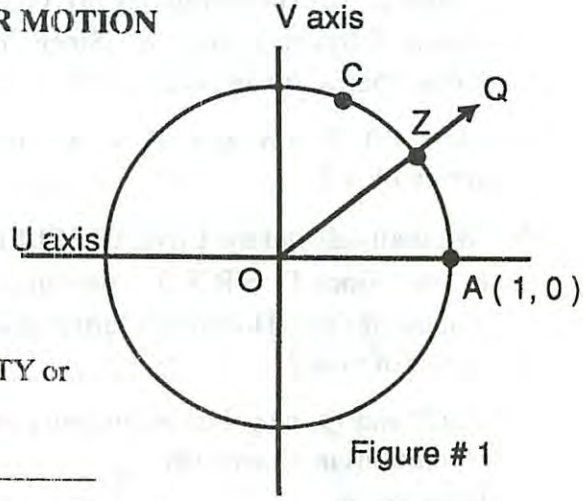


Figure # 1

Z is a point on the unit circle in figure # 1. The ray OQ rotates with uniform speed, so Z moves around the circle with uniform speed. Let T be the time required for the ray to complete 1 revolution.

(1) How far does Z travel in one revolution ? 2π

(2) How fast is Z traveling ? $\frac{2\pi}{T}$

(3) We denote the speed of Z ON THE UNIT circle by ω. ω is the greek letter omega. ω is called the ANGULAR VELOCITY or ROTATIONAL VELOCITY.

How far can Z travel in t units of time ? $\omega t = \frac{2\pi}{T} t$

(4) C is a fixed point on the unit circle. If it takes t seconds for Z to travel from the (also) fixed point A to C, how far has Z traveled? ωt

(5) What are the coordinates of Z when Z is at point C ? $(\cos \omega t, \sin \omega t)$

(6) Z moves in a counter clockwise manner starting at A when t = 0. Z completes 2 revolutions in one second. What is T ? $\frac{1}{2}$ Sec What is ω ? 4π

(7) If Z completes 60 revolutions in one second then What is T ? $\frac{1}{60}$ What is ω ? 120π

(8) If Z completes 4 revolutions in three seconds, then What is T ? $\frac{3}{4}$ What is ω ? $\frac{8\pi}{3}$

(9.) If $\omega = 120\pi$ per second and Z starts at (1 , 0) when t = 0 , then what are the coordinates of Z at $t = 2$?
U = $\cos 240\pi$, V = $\sin 240\pi$ = (1 , 0)

(10) If $\omega = \frac{8\pi}{3}$ per second and Z starts at (1 , 0) when t = 0. When t = 1 what are the coordinates of Z ?

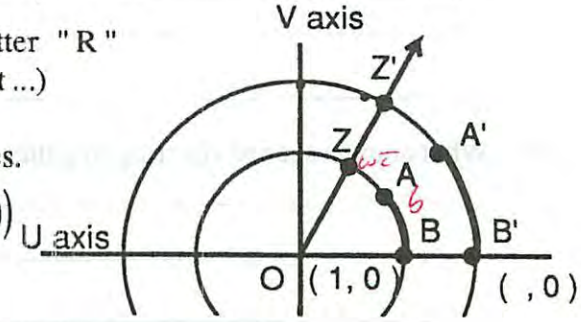
U = Cos $\frac{8\pi}{3}$ = Cos $\frac{2\pi + \frac{2\pi}{3}}{}$ = Cos $\frac{2\pi}{3}$ = $-\frac{1}{2}$

V = Sin $\frac{8\pi}{3}$ = Sin $\frac{2\pi + \frac{2\pi}{3}}{}$ = $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ so coordinates are : $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(11) In Figure # 2 B = (1, 0) ; ArcAB = b . The arc length from A to Z is ωt . That is, Z started at A with rotational velocity ω . The coordinates of Z are U = $\cos(\omega t + b)$, V = _____

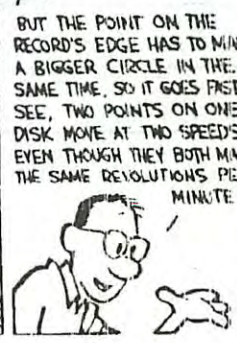
(12) We use the greek letter ρ , which looks like a " P " but is the letter " R " and is pronounced " ROW ". (like in Row, Row, Row your boat ...) to represent the radius of a non-unit circle.
Since angle BOZ = angle B'OZ' we have similar triangles.

The coordinates for Z' are $(\rho \cos(\omega t + b),\ \rho \sin(\omega t + b))$



Notice that the angle is based on the unit circle !

The radius adjustment is made ONLY with ρ .



(13) Let's try to help Calvin get to sleep.
Z and Z' are on the rotating ray OQ which is turning at a rate of ω. That is Z travels at rate ω. Since $\rho > 0$ it is clear that Z' travels farther than Z for any time $t > 0$.

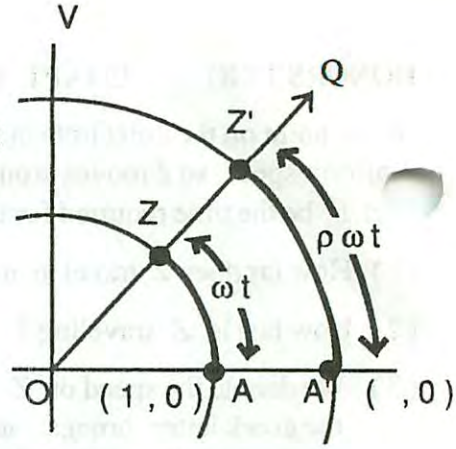


Figure # 3

(14) At $t = 0$, $Z = A$ and $Z' = A'$. How far do Z and Z' travel in terms of t ? ωt $\rho \omega t$

(15) We shall call $\rho\omega$ the LINEAR SPEED. Thus, the the velocity of Z' is $\rho\omega$. Since D = R X T, we can say that Z' travels $\rho\omega t$ in t units of time. How much farther does Z' travel than Z for t units of time ? $\rho\omega t - \rho\omega = (\rho - 1)\omega t$

(16) Let P and Q be points on the unit circle in figure # 4. P makes 2 revolutions in 3 seconds. Q makes 3 revolutions in 12 seconds.
What are ω_P $\frac{4\pi}{3}$ and ω_Q $\frac{\pi}{2}$?

(17) P and Q start at A. You should expect P or Q to lap P or Q for some time $t > 0$. When this event takes place, how much farther will P have traveled ? _____

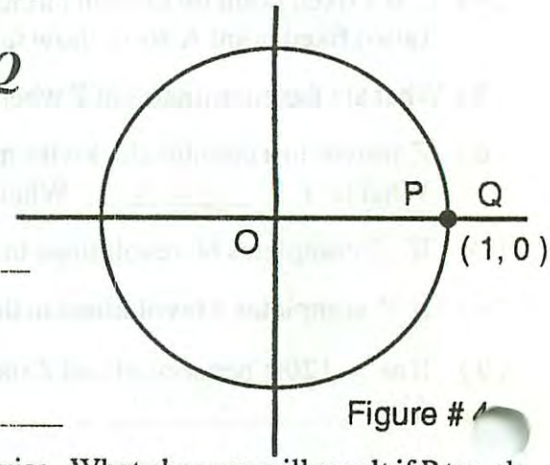


Figure # 4

(18) Distance for P = (Distance for Q) + 2π.
This translates in what equation ? _____
Solve for t. _____.

The significance of this is that this is the time required for
_____

(19) Above in (16) - (18) we assumed that P and Q traveled counter clockwise. What changes will result if P travels clockwise and Q travels counter clockwise? Speculate ...

_____

_____

_____

_____

_____

_____

20) What about a second meeting in either problem 19 or the previous setting?

_____

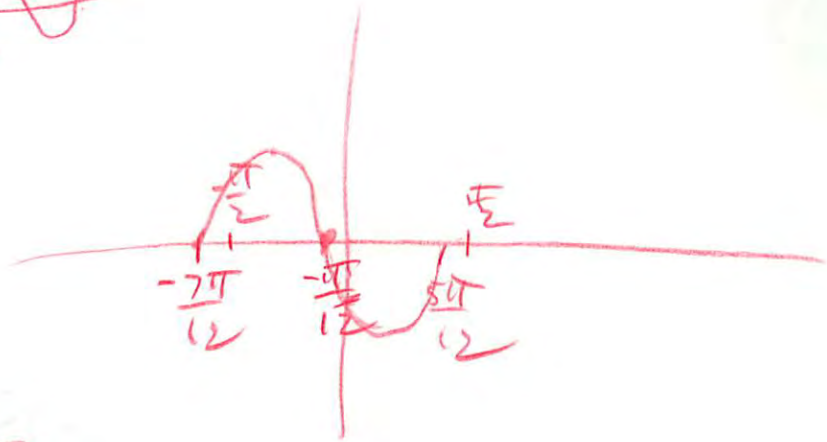
_____

_____

_____

_____

$\cos 2(x+\frac{\pi}{3})$



$$2(x+\frac{\pi}{3}) \overset{\text{set}}{=} k\pi + \frac{\pi}{2}$$

$$x+\frac{\pi}{3} = \frac{k\pi}{2} + \frac{\pi}{4}$$

$$x = \frac{k\pi}{2} + \frac{3\pi}{12} - \frac{4\pi}{12}$$

$$= \frac{(6k-1)\pi}{12}$$

P.S. not necessarily the same thing as head start because P.S. is related to ω (ie speed and period length) while head start is completely independent of speed, period etc

items 11 & 12 on uniform circular motion

Ditto says ...

$$\rho\cos(\omega t+b),\ \rho\sin(\omega t+b)$$

where b is "head start" as measured on the unit circle.

This should <u>NOT</u> be confused with phase shift.

Phase shift has to do with horizontal shift of the <u>graph</u> of a fctn. "Head start" has to do with movement from the "standard starting position" of $(\rho, 0)$.

Thus..., in $(\rho\cos(\omega t+b), \rho\sin(\omega t+b))$, the head start is b. The phase shift is ~~$\rho(\omega t+b)$~~

~~So~~ is the relationship found ....

$$\left(\rho\cos\omega\left(t+\frac{b}{\omega}\right), \rho\sin\omega\left(t+\frac{b}{\omega}\right)\right)$$

$$\rho\cos(\omega t+b),\ \rho\cos\omega\left(t+\frac{b}{\omega}\right)$$

by $\frac{b}{\omega}$

$\frac{b}{\omega}$ is the phase shift when b is head start.

So when $\omega=1$, phase shift = head start = "b" number.

(15) $P_1\ (3, 0)$: $P_2\ \left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

$\omega_1 = \frac{\pi}{3}$ $\qquad$ $\omega_2 = \frac{\pi}{4}$

faster

dist fast = dist slow + head start

$$3 \cdot \frac{\pi}{3} t = 3\frac{\pi}{4} \cdot t + 3 \cdot \frac{\pi}{6}$$

$$t = \frac{3t}{4} + \frac{1}{2}$$

$$\frac{1}{4} t = \frac{1}{2}$$

$$t = 2$$

coordinates

$3\cos\left(\frac{\pi}{3} \cdot 2\right), 3\sin\left(\frac{\pi}{3} \cdot 2\right)$

$\left(3\cos\frac{2\pi}{3}, 3\sin\frac{2\pi}{3}\right)$

$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

$3\cos\left(\frac{\pi}{4} \cdot 2 + \frac{\pi}{6}\right), 3\sin\left(\frac{\pi}{4} \cdot 2 + \frac{\pi}{6}\right)$

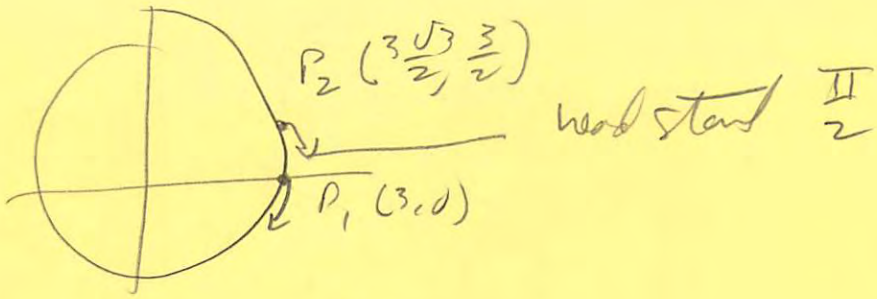
$3\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right),$

$3\cos\left(\frac{4\pi}{6}\right),$

$\left(3\cos\frac{2\pi}{3}, 3\sin\frac{2\pi}{3}\right)$

$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

#16



$\omega_1 = \frac{-\pi}{3}$ $\quad$ $\omega_2 = -\frac{\pi}{4}$

1 full Lap is 6π

dist fast = dist slow + extra

$3 \cdot \frac{\pi}{3} t = 3 \frac{\pi}{4} t + \frac{11\pi}{2}$

$\frac{t}{4} = \frac{11}{2}$

$t = 22$

$\left(3\cos 22\left(-\frac{\pi}{3}\right),\right.$

$3\cos\frac{22\pi}{3}$

$3\cos\left(\frac{18\pi}{3} + \frac{4\pi}{3}\right)$

$3\cos\frac{-4\pi}{3},\ 3\sin\frac{-4\pi}{3}\Big)$

$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

$3\cos\left(22\left(-\frac{\pi}{4}\right) + \frac{\pi}{6}\right.$

$3\cos\left(-\frac{11\pi}{2} + \frac{\pi}{6}\right)$

$3\cos\left(-\frac{33+1}{6}\right.$

$3\cos\left(-\frac{32\pi}{6}\right.$

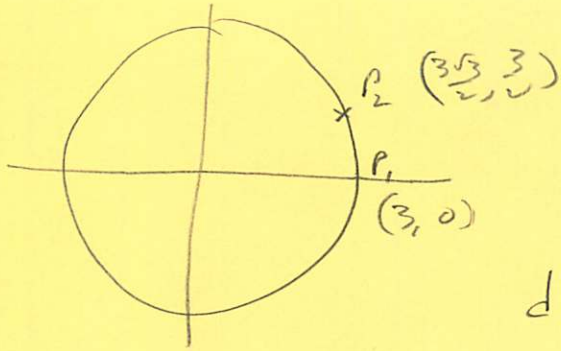
$3\cos\left(-\frac{16\pi}{3}\right)$

$3\cos\left(-\frac{12\pi}{3} - \frac{4\pi}{3}\right)$

$3\cos\left(-\frac{4\pi}{3}\right)$

$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

(15)



$\rho = 3$

$\omega_1 = \frac{\pi}{3}$ $\omega_2 \frac{\pi}{4}$

$$d P_1 = d P_2 + 3 \cdot \frac{\pi}{6}$$

$$3 \cdot \frac{\pi}{3} \cdot \tau = 3 \cdot \frac{\pi}{4} \cdot \tau + \frac{\pi}{2}$$

$$\tau = \frac{3}{4}\tau + \frac{1}{2}$$

$$\frac{1}{4}\tau = \frac{1}{2}$$

$$\tau = 2$$

Point 1

$3 \cos \frac{\pi}{3} \cdot \tau, \; 3 \sin \frac{\pi}{3} \tau$

$3 \cdot \cos \frac{2\pi}{3}, \; 3 \sin \frac{2\pi}{3}$

$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

$\rho \omega \tau$

$3 \cdot \frac{\pi}{3} \cdot 2$

2π

Point 2

$\left(3 \cos \frac{\pi}{4} \cdot 2 + \frac{\pi}{6}, \; 3 \sin \frac{\pi}{4} \cdot 2 + \frac{\pi}{6}\right)$

$3 \cos \frac{\pi}{2} + \frac{\pi}{6}, \; 3 \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$

$\left(3 \cos \frac{2\pi}{3}, \; 3 \sin \frac{2\pi}{3}\right)$

$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

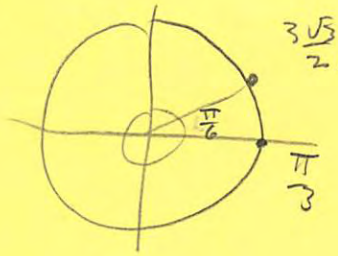
$\rho \omega \tau$

$3 \cdot \frac{\pi}{4} \cdot 2$

$\frac{3\pi}{2}$

Pg 99

(15)



$df = ds + 3 \cdot \frac{\pi}{6}$

$3\frac{\pi}{3}t = 3\frac{\pi}{4}t + \frac{\pi}{2}$

$\frac{t}{4} = \frac{1}{2}$

$t = 2$

$3\cos\frac{\pi}{3}\cdot 2,\ 3\sin\frac{\pi}{3}\cdot 2$

$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

$3\cos\frac{\pi}{4}\cdot 2 + \frac{\pi}{6},\ 3\sin\frac{\pi}{4}\cdot 2 + \frac{\pi}{6}$

$3\cos\frac{\pi}{2} + \frac{\pi}{6},$

$\left(3\cos\frac{2\pi}{3}, 3\sin\frac{2\pi}{3}\right)$

(16)

$df = ds + \frac{11\pi}{2}$

$3\frac{\pi}{3}t = 3\frac{\pi}{4}t + \frac{11\pi}{2}$

$\frac{t}{4} = \frac{11}{2}$

$t = 22$

$\left(3\cos\frac{-\pi}{3}\cdot 22, 3\sin\frac{-22\pi}{3}\right)$

$\left(3\cos\frac{-18\pi}{3} - \frac{4\pi}{3}, 3\sin\frac{-4\pi}{3}\right)$

$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

$3\cos\left(\frac{-\pi}{4}\cdot 22 + \frac{\pi}{6}\right), 3\sin\left(-\frac{11\pi}{2} + \frac{\pi}{6}\right)$

$3\cos\left(\frac{-32\pi}{6}\right)\ 3\sin\left(\frac{-16\pi}{3}\right)$

$3\cos\left(\frac{-12\pi}{3} - \frac{4\pi}{3}\right), 3\sin\frac{-4\pi}{3}$

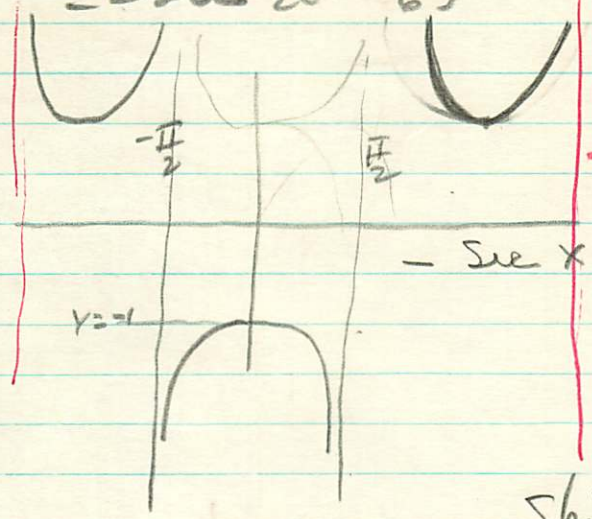
$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

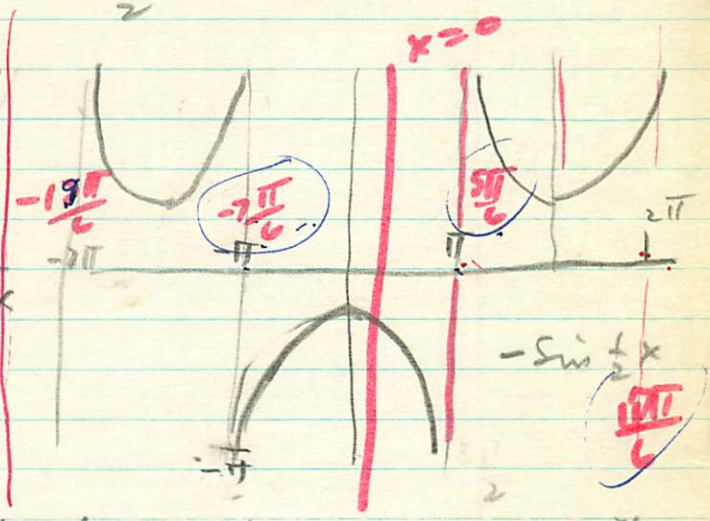
Example 2

$y = -\sec\left(\frac{1}{2}x + \frac{\pi}{12}\right)$

$= -\sec \frac{1}{2}\left(x + \frac{\pi}{6}\right)$

$\frac{2\pi}{\frac{1}{2}} = 4\pi = \text{Period}$





Shift Graph to Left Since $\frac{\pi}{6} > 0$

To compute asymptotes

$\sec x \Rightarrow x \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

$\sec \frac{1}{2}x \Rightarrow \frac{1}{2}x \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

$x \neq \left\{(2k+1)\pi\right\}$

$\sec\left(\frac{1}{2}x + \frac{\pi}{12}\right) \Rightarrow \frac{1}{2}x + \frac{\pi}{12} \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

$\frac{1}{2}x \neq \left\{\frac{(2k+1)\pi}{2} - \frac{\pi}{12}\right\}$

$x \neq \left\{(2k+1)\pi - \frac{\pi}{6}\right\}$

$\left\{\frac{6(2k+1)\pi - \pi}{6}\right\}$

$\left\{\frac{12k\pi + 6\pi - \pi}{6}\right\}$

$\left\{\frac{(12k+5)\pi}{6}\right\}$

$-19 \quad -13 \quad -7$

$a_n = a_1 + (n-1)d$

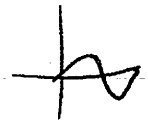
$a_n = -19 + 6n - 6$

$= \frac{-19 + 6n}{6}$

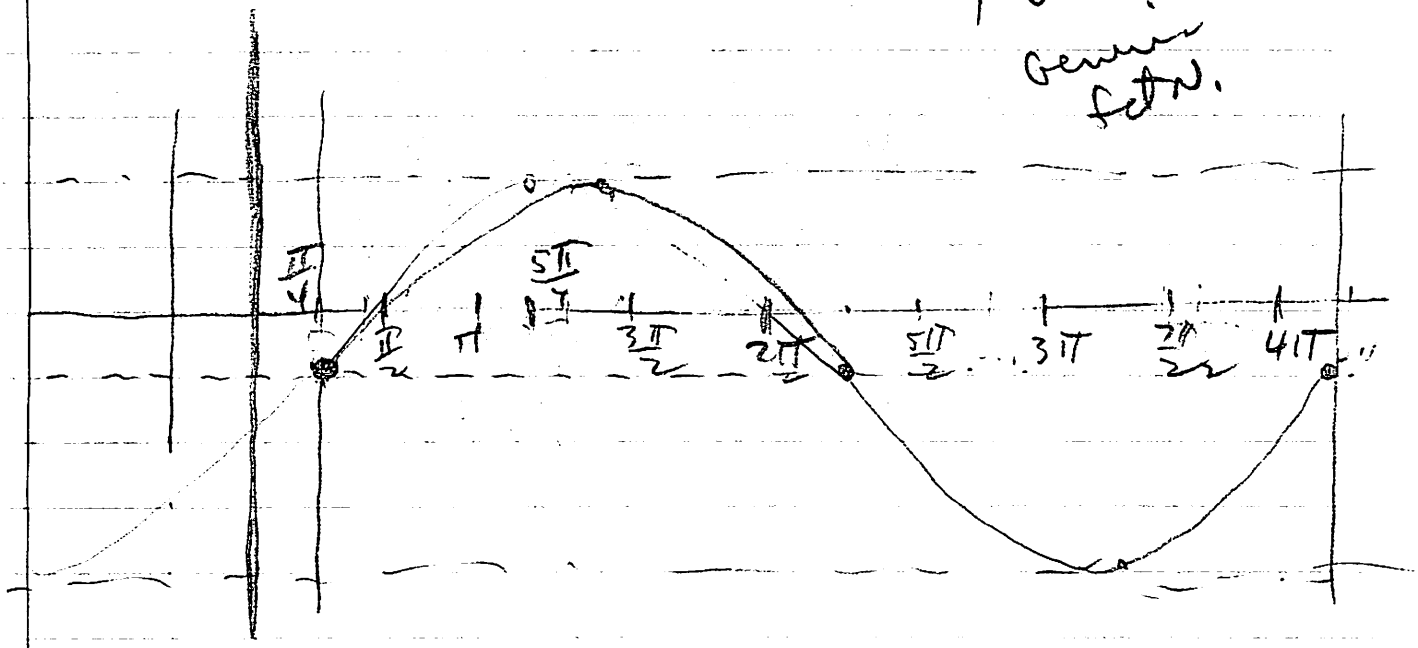
$\frac{-19 + 6k}{6}$

"Box"

$f(x) = 3\sin\frac{1}{2}\left(x - \frac{\pi}{4}\right) - 1$



Genuine fctn.



1. Dot in center line
2. Dot in max & min. (Amplitude)
3. Get new asymptotes or zeros.

$$\frac{1}{2}\left(x - \frac{\pi}{4}\right) = k\pi$$

$$x - \frac{\pi}{4} = 2k\pi$$

$$x = 2k\pi + \frac{\pi}{4}$$

$$x = \frac{(8k+1)\pi}{4}$$

Dot in zeros on "axis of fctn"

Draw "Box"

4. $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$

5. Back up so curve crosses Y axis.

$\cos^{-1}\left(\frac{4}{3}x - \frac{2\pi}{9}\right)$

$\frac{4}{3}\left(x - \frac{\pi}{6}\right)$

$\frac{4}{3}\left(x - \frac{\pi}{6}\right) = \frac{(2k+1)\pi}{2}$

$4\left(x - \frac{\pi}{6}\right) = \frac{6k\pi}{2} + \frac{3\pi}{2}$

$x - \frac{\pi}{6} = \frac{6k\pi}{8} + \frac{3\pi}{8}$

$x = \frac{18k\pi}{24} + \frac{9\pi}{24} \quad \frac{4\pi}{24}$

$\frac{(18k+13)\pi}{24}$

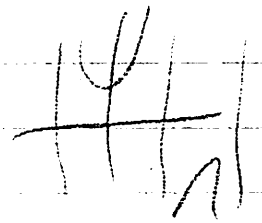
$\frac{-5\pi}{24}$

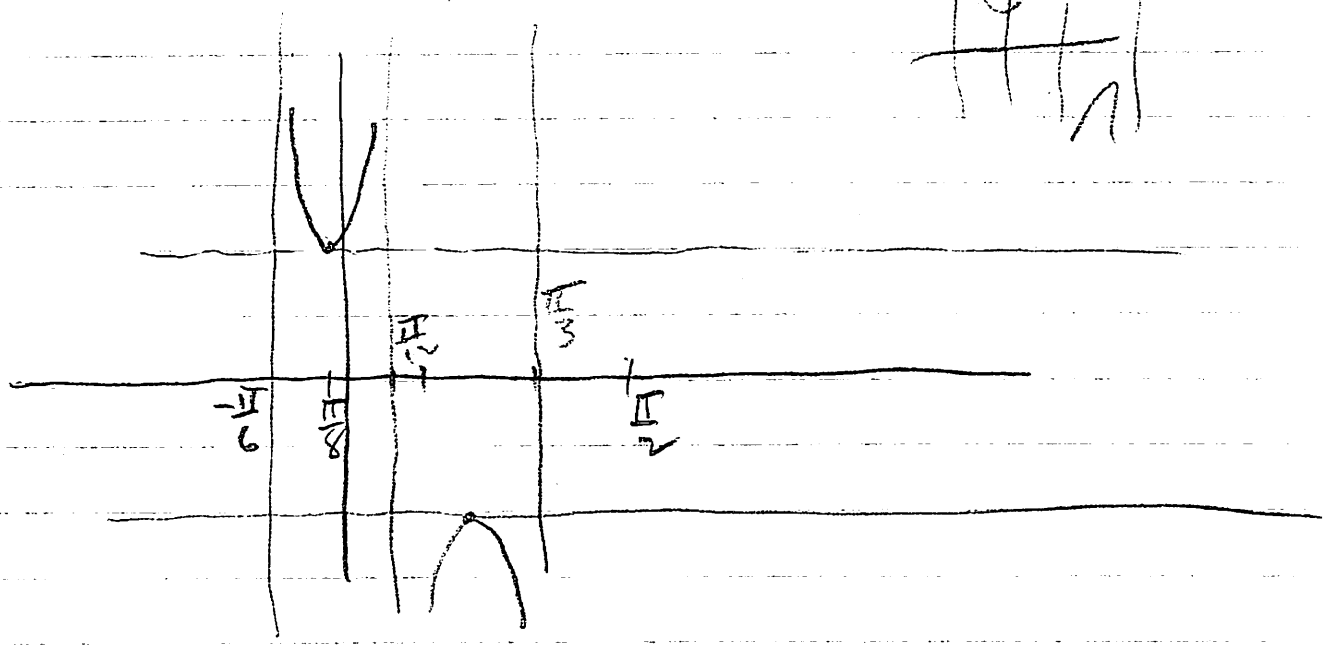
$\frac{13\pi}{24}$

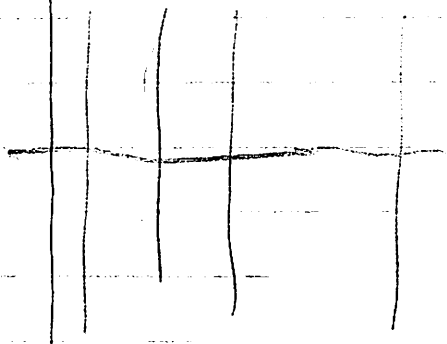
$\frac{3\pi}{24}$

$f(x) = -\sec\left(2x + \frac{\pi}{3}\right)$

$-\sec 2\left(x + \frac{\pi}{6}\right)$







$K = -1$

$$2\left(x + \frac{\pi}{6}\right) = \frac{(2K+1)\pi}{2}$$

$$x + \frac{\pi}{6} = \frac{K\pi}{4} + \frac{\pi}{4}$$

$$x = \frac{K\pi}{4} + \frac{\pi}{12}$$

$$x = \frac{(3K+1)\pi}{12}$$

$\frac{3\pi}{12}$

$\frac{3\pi}{24}$

$2 + 4$

6

QUIZ #1 CHAPTER 3

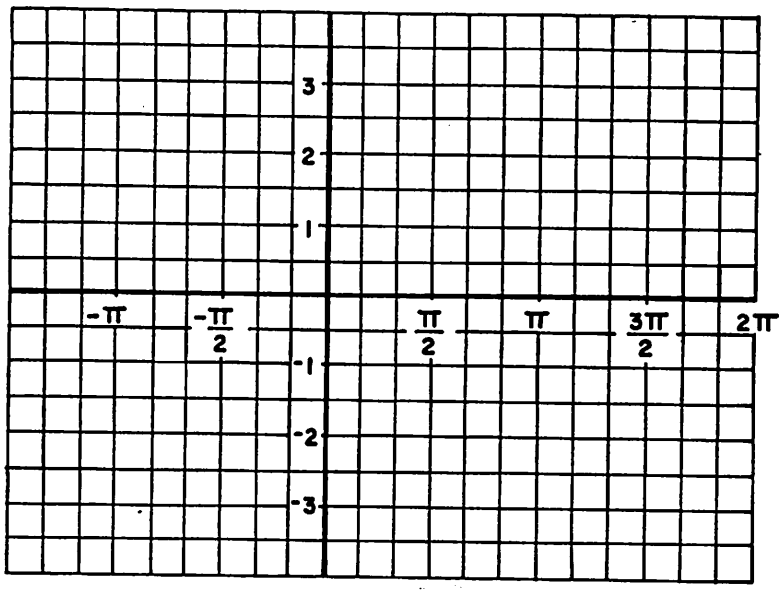
Name _____

Period _____

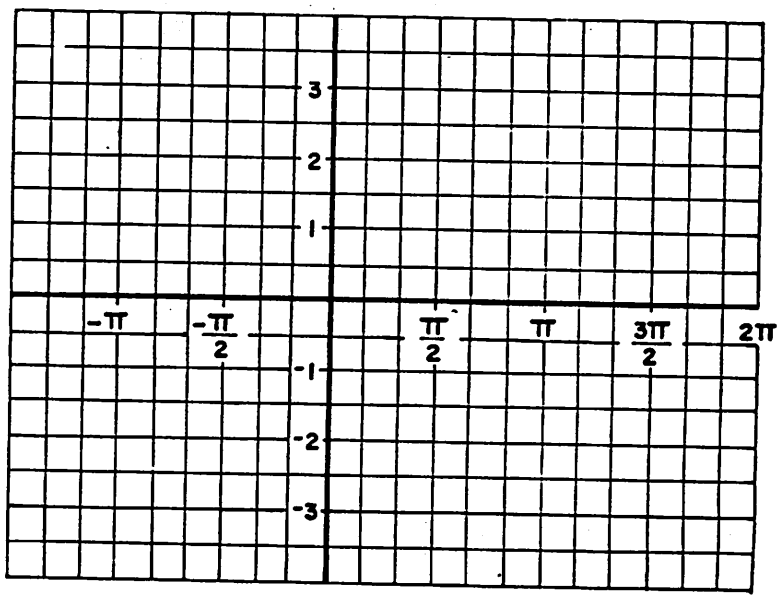
Graph the following functions. Mark your curves with the COORDINATES of the maximum and minimum points. State the domain, range, amplitude, period and phase shift (in the proper direction). Draw at least one period.

(10 pts ea)

A) $F(x) = 3 \cos x + 1$



B) $G(x) = \sin\left(2x + \frac{\pi}{2}\right) - 1$



QUIZ #1 CHAPTER 3

Name 10/20/86

Period _____

Graph the following functions. Mark your curves with the COORDINATES of the maximum and minimum points. State the domain, range, amplitude, period and phase shift (in the proper direction). Draw at least one period.

(10 pts ea)

A) $F(x) = 3 \cos x + 1$

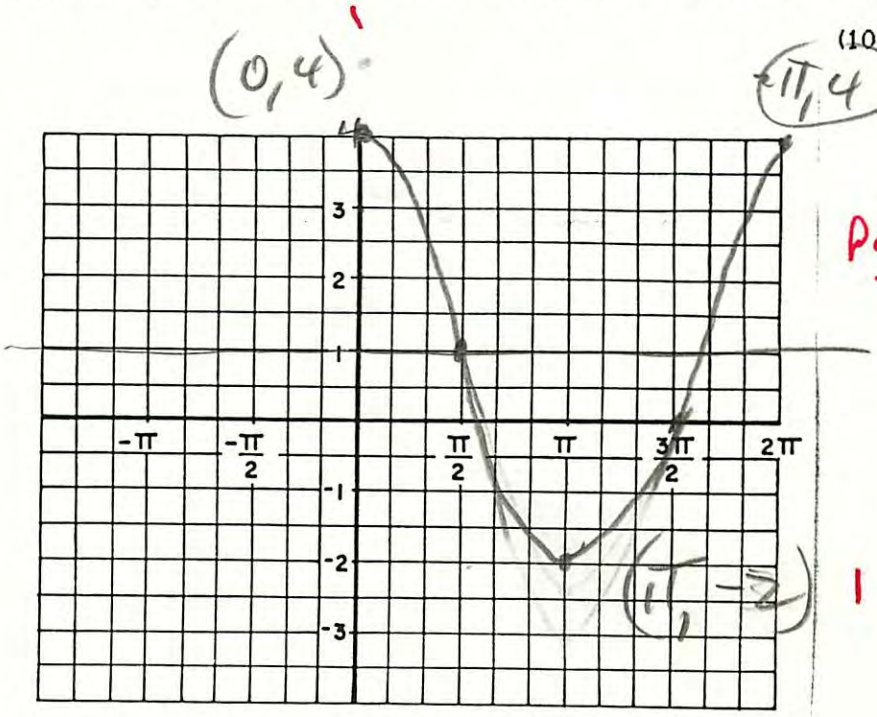
1 D_f $x \in \mathbb{R}$

1 R_f $-2 \le f(x) \le 4$

1 Amp 3

1 Per 2π

1 PS None



Pict 3.

B) $G(x) = \sin\left(2x + \frac{\pi}{2}\right) - 1$

$\sin 2\left(x + \frac{\pi}{4}\right) - 1$

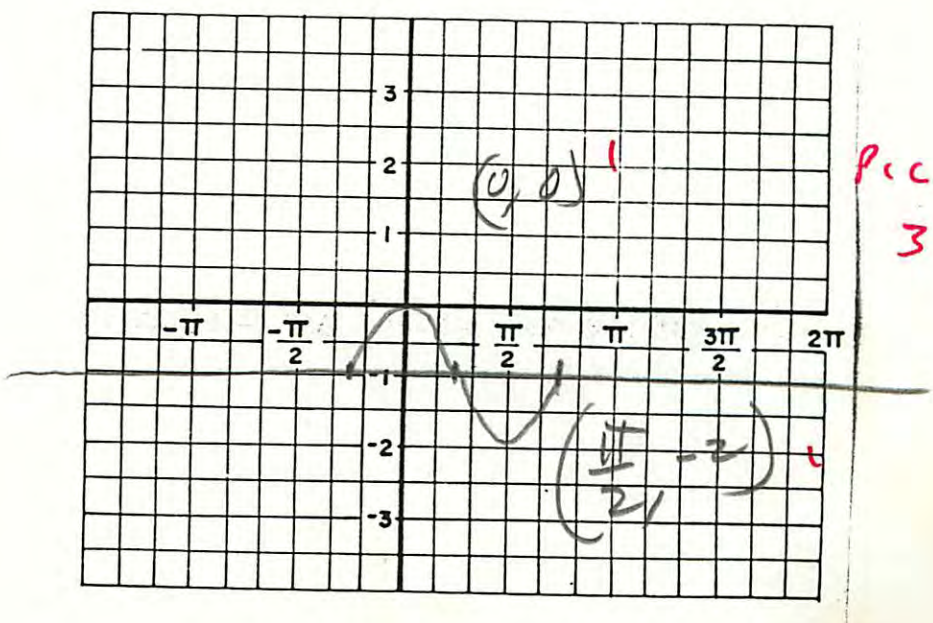
1 D_g $x \in \mathbb{R}$

1 R_g $-2 \le g(x) \le 0$

1 Amp 1

1 Per π

1 PS $\frac{\pi}{4}$ Left



Pic 3

QUIZ #2 CHAPTER 3

Name _____

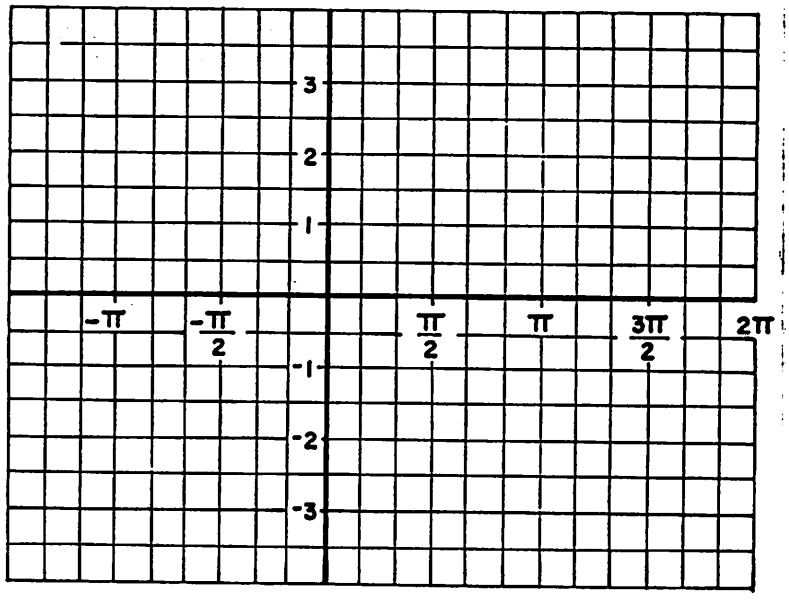
Period _____

A) Graph the given function. Mark your curve with the COORDINATES of the maximum and minimum points. State the domain, range, amplitude, period and phase shift (in the proper direction). Draw at least one period.

Locate and mark on your graph the point (x , F(x)) when $x = \frac{5\pi}{4}$. Using the proper addition formula and simplyfing F(x), demonstrate that your drawing is correct.

(10 pts total)

$F(x) = 2 \text{ Sin } \left(\frac{2x}{3} + \frac{\pi}{6} \right)$



B) State the domain, range, amplitude, period and phase shift (in the proper direction) for each of the following. Do NOT draw the graphs. (2 pts ea)

1) $A(x) = 3 \text{ Sin } 2x$

2) $B(x) = 2 \text{ Sec } x + 1$

3) $C(x) = 3 \text{ Tan } \frac{3x}{2}$

4) $D(x) = \frac{1}{2} \text{ Cos } \left(6x - \frac{\pi}{3} \right)$

5) $E(x) = \frac{1}{2} \text{ Csc } 2x + 1$

QUIZ #2 CHAPTER 3

$2\sin\frac{2}{3}\left(\frac{5\pi}{4}+\frac{\pi}{4}\right)-3$

$2\sin\frac{2}{3}\left(\frac{3\pi}{2}\right)-3$

$2\sin\pi-3$

-3

1

Name _____

Period _____

18 ↑ 16 +
16 - 17 ✓
14 - 15 0
10 - 13 −
less 10 =

A) Graph the given function. Mark your curve with the COORDINATES of the maximum and minimum points. State the domain, range, amplitude, period and phase shift (in the proper direction). Draw at least one period.

Locate and mark on your graph the point (x , F(x)) when $x = \frac{5\pi}{4}$. Using the proper addition formula and simplyfing F(x), demonstrate that your drawing is correct.

(10 pts total)

removed

$F(x) = 2\sin\left(\frac{2x}{3} + \frac{\pi}{6}\right) - 3$

$= 2\sin\frac{2}{3}\left(x+\frac{\pi}{4}\right)-3$

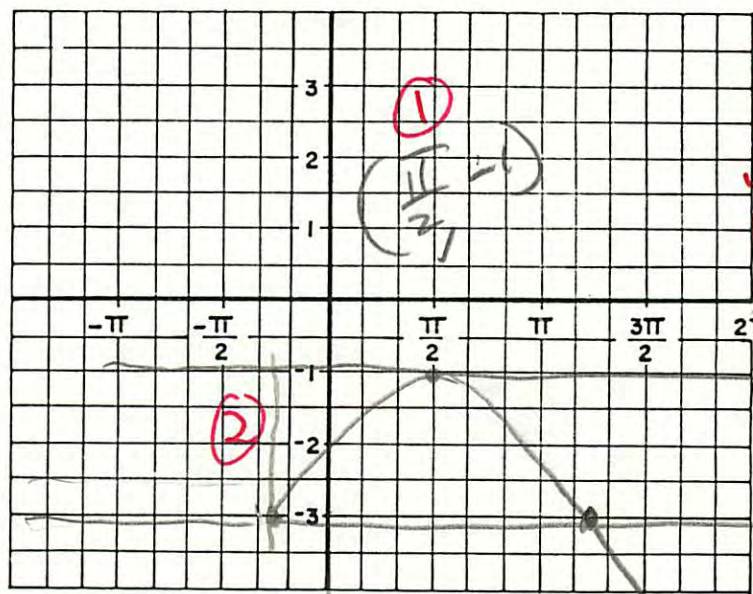
$\frac{2\pi}{\frac{2}{3}} = 3\pi$

$0 \to -\frac{\pi}{4}$ $\quad \frac{3\pi}{4} \to \frac{\pi}{2}$

$\frac{3\pi}{2} \to \frac{5\pi}{4}$ $\quad \frac{11\pi}{4}$

$3\pi \to 2\frac{3}{4}\pi$ $\quad 2\pi$

$\frac{3}{4}\cdot 3\pi$ $\quad \frac{9\pi}{4}$



D_f $x \in \mathbb{R}$

R_f $-5 \le f(x) \le 1$

Amp = 2

Pu = 3π

PS = $\frac{\pi}{4}$ Left

1

$(2\pi, -5)$

$\frac{11\pi}{4}$

B) State the domain, range, amplitude, period and phase shift (in the proper direction) for each of the following. Do NOT draw the graphs. (2 pts ea)

$\frac{3}{2}x = \frac{2k+1}{2}\pi$

2 pt ea No Partial credit

1) $A(x) = 3\sin 2x$

D_A $x \in \mathbb{R}$

R_A $|A(x)| \le 3$

Pu π

PS None

$6\left(x - \frac{\pi}{18}\right.$

2) $B(x) = 2\sec x + 1$

D_B $x \notin \frac{(2k+1)\pi}{2}$

R_B $B(x) \ge 3$ or $B(x) < -1$

Amp = 2

Pu 2π

PS None

3) $C(x) = 3\tan\frac{3x}{2}$

D_B $x \notin \frac{(2k+1)\pi}{3}$

R_B $B(x) \in \mathbb{R}$

Pu $\frac{2\pi}{3}$

PS None

$2x \notin k\pi$

4) $D(x) = \frac{1}{2}\cos\left(6x - \frac{\pi}{3}\right)$

D_D $x \in \mathbb{R}$

R_D $|D(x)| \le \frac{1}{2}$

Pu $\frac{\pi}{3}$

PS $\frac{\pi}{18}$ Right

5) $E(x) = \frac{1}{2}\csc 2x + 1$

D_E $x \notin \left\{\frac{k\pi}{2}\right\}$

R_E $E(x) \ge \frac{3}{2}$ or $E(x) \le \frac{1}{2}$

Pu π

PS None

QUIZ #3 CHAPTER 3

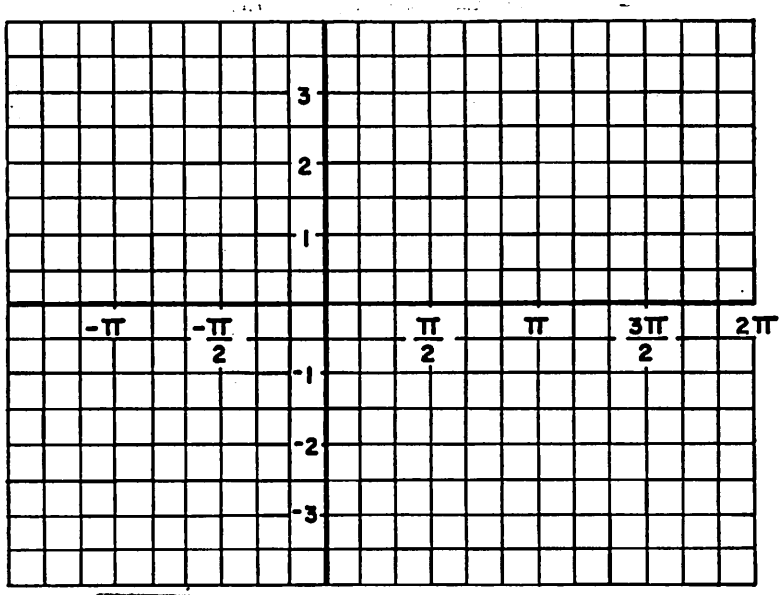
Name _____

Period _____

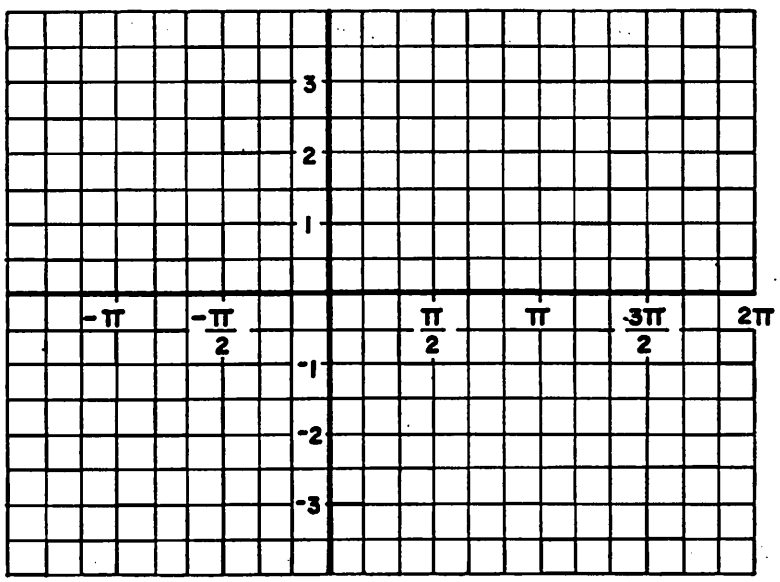
Graph the following functions.
State the domain, range, amplitude, period and phase shift (in the proper direction).

(10 pts ea)

A) $F(x) = -\operatorname{Sec}\left(\frac{1}{2}x + \frac{\pi}{12}\right)$



B) $G(x) = 2\operatorname{Tan} x + 1$



QUIZ #3 CHAPTER 3

Name _____

Period _____

Graph the following functions.
State the domain, range, amplitude, period and phase shift (in the proper direction).

(10 pts ea)

A) $F(x) = -\text{Sec}\left(\frac{1}{2}x + \frac{\pi}{12}\right)$

$-\sec\frac{1}{2}\left(x+\frac{\pi}{6}\right)$

$\frac{1}{2}x+\frac{\pi}{12} = \frac{(2k+1)\pi}{2}$

$6x+\pi = 12k\pi + 6\pi$

$6x = 12k\pi + 5\pi$

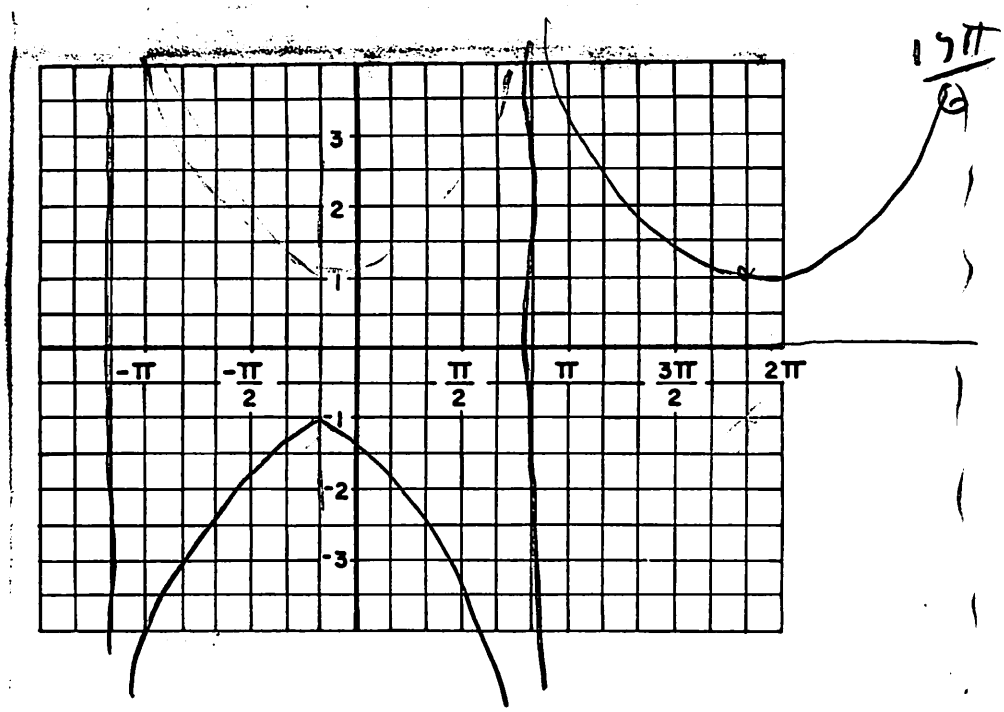
$x = \frac{(12k+5)\pi}{6}$

Per 4π

Amp $= -1$

PS $\frac{\pi}{6}$ Left

R $f(x) \geq 1$ or $f(x) \leq -1$



B) $G(x) = 2\,\text{Tan}\,x + 1$

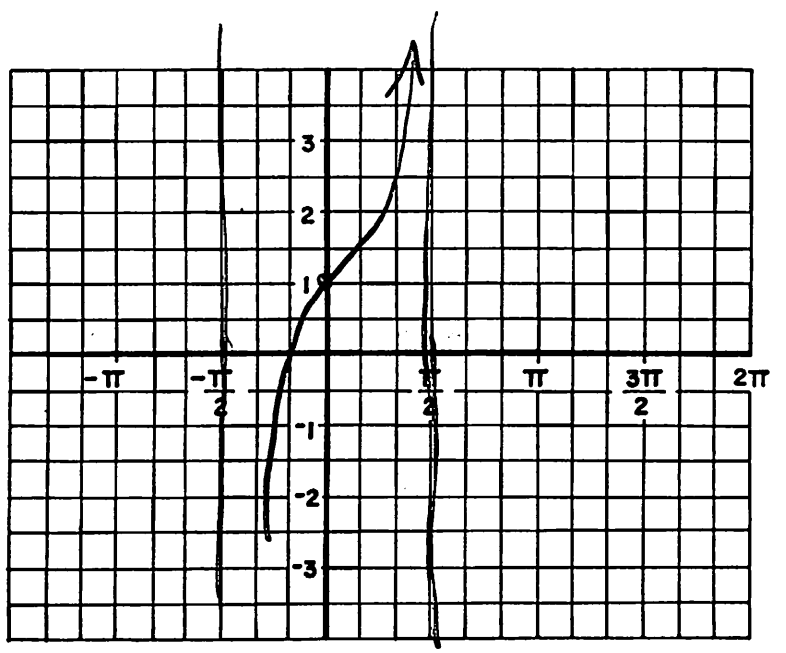
D $x \neq \frac{(2k+1)\pi}{2}$

R $G(x) \in \mathbb{R}$

Amp $= 2$

Per π

PS none



Honors Trig
Chapter 3
Mr. Mumaugh

WEEKEND QUICKIE

I Horatio Hypochondria has been absent from school so long that even he has forgotten where his desk is in the classroom. Horatio called ***you*** about 6:30 P.M. (Horatio, as we all know, has more brass than a band.) and begged you to "catch him up" on what he has missed in your favorite class, **TRIGONOMETRY**. Horatio seems to have forgotten everything! You start to panic and then remember that *your* awesome trig expertise will save the day for Horatio. ***You can*** change Horatio from a lowly pretender to a worthy contender! You quickly jot down the following list of topics.
You will turn in your written (and very explicit answers) to the following questions. Include drawings when they will help to clarify a point you are making but remember that pictures are not to take the place of explanations! This homework is worth 5 regular assignments. ***Only the very best and most complete responses should expect pluses.***

A) Explain *everything there is to know* about a, b, c, and d in : $a\,\mathrm{Fctn}\,(bx - c) + d$.

B) Explain how knowing which intervals on the x-axis correspond to which quadrants on the unit circle helps in drawing the graph of *any* trig function.

C) Explain how we use the "asymptotes" method to draw some trig graphs.

D) Explain how the "asymptote method" and the "zeros method" are similar and how they are different and when you and why you would use the "zeros method."

E) Explain what a "generic" picture is and how we use it.

F) Explain what "omega" is. Explain what " rho " is. Explain what the "argument" is. Explain how we determine the NEW period length.

G) Explain the difference between rotational , linear, and angular velocities.

H) Explain, in words, how to properly graph the function: $A(x) = 2\sin\left(\frac{2}{3}x - \frac{\pi}{6}\right)$

I) Explain ***EVERYTHING*** there is to know about two points traveling around a non-unit circle at different speeds (and directions).

II Draw the graph of $f(x)$. Be sure your method is VERY clear to Horatio (and me).

A) $f(x) = 2\sin x + \cos x$ B) $f(x) = 2\sin x \cos x$ C) $f(x) = \sqrt{\sin^2 4x + \cos^2 4x}$

D) $f(x) = \cos\frac{1}{2}x + 2\sin\frac{3}{2}x$ E) $f(x) = \csc^2 x - \cot^2 x$

III p_1 starts at (3, 0) and p_2 starts at $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$, $\omega_1 = -\frac{\pi}{3}$ and $\omega_2 = -\frac{\pi}{4}$.

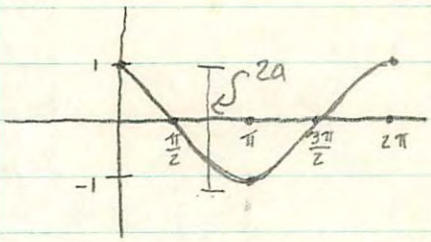
A) When will the two points coincide for the first time after $t = 0$? Use ω_1 to compute the coordinates for that first meeting. Next use ω_2 to compute the coordinates. They are the same point aren't they?

B) When AND where will the two points coincide for the **third** time after $t = 0$?

IV Close your phone call from Horatio with a "Thank you." because now **you** are ready for your 3 trig exam!

3-1 Pg 97-102 Graph of Cos + Sin.

using a table of values

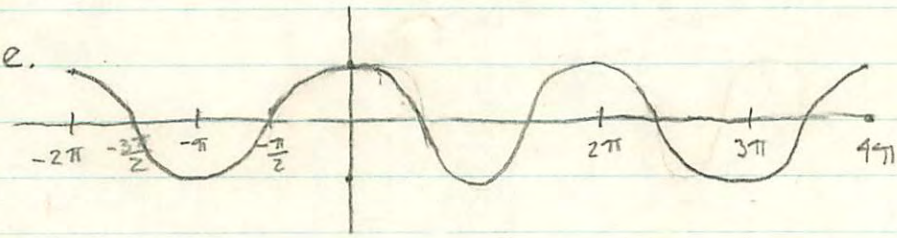


x	$\cos x$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

a smooth rounded curve amplitude means the amount of fluctuation in the curve. Amplitude is $\frac{1}{2}$ distance between max and min pts on curve. Cos goes between 1 and -1 or distance of $2 \cdot \frac{1}{2} \cdot 2 = 1$ Thus Cos has an amplitude of 1.

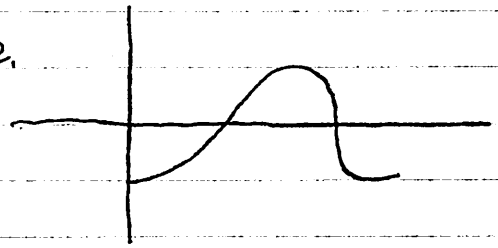
The Cos is periodic w/period 2π which means what I have drawn is on full period. If I want to see beyond 2π or less than 0, I simply repeat exactly what I have drawn

i.e.

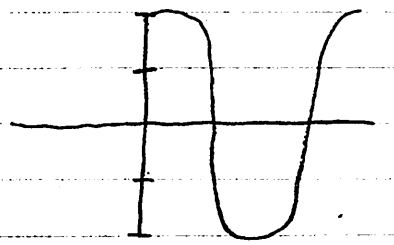


If the equation was $y = -\cos x$, I have the same equation inverted.

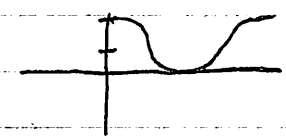
ie.



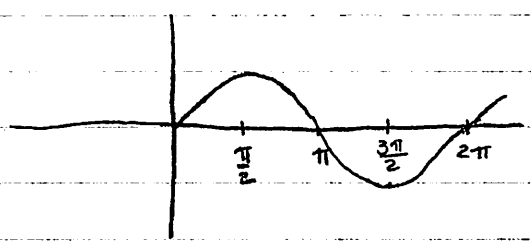
If the equation was $y = 2\cos x$ the amplitude has been increased from 1 to 2:



If $y = \cos x + 1$ we simply raise the ~~amplitude~~ curve 1 unit

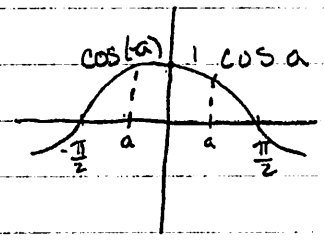


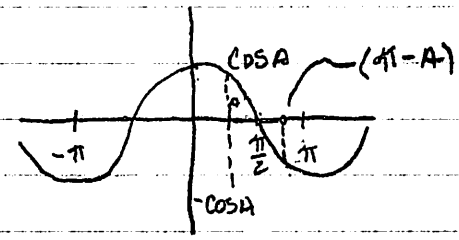
The sin fctn. acts in a similar manner. using a table of values to find the graph of sin x we have:



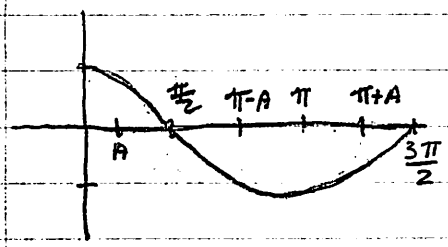
x	$\sin x$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

3-1 Pg 101

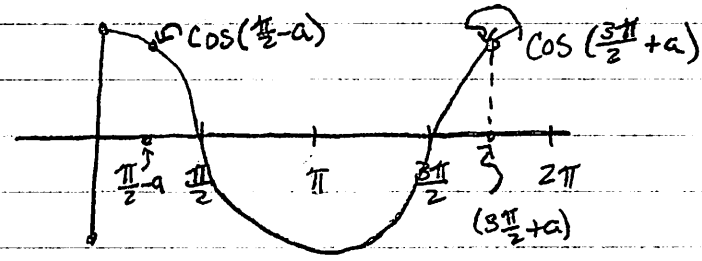
① 

② 

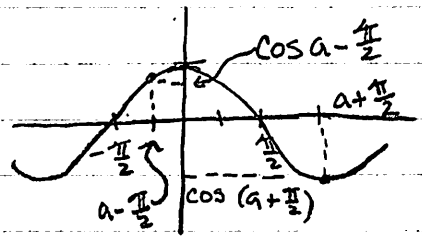
③ $\cos(\pi+A) = \cos(\pi-A)$



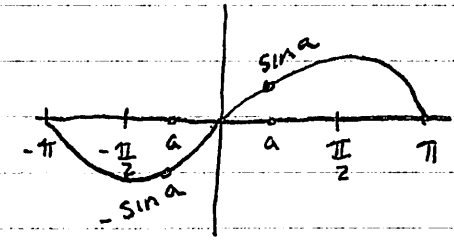
⑤ $\cos\left(\frac{\pi}{2}-a\right) = \cos\left(\frac{3\pi}{2}+a\right)$



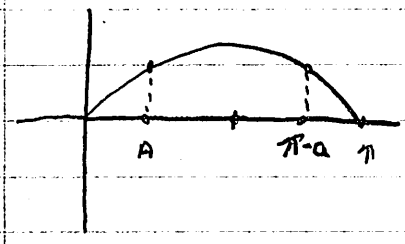
⑦ $\cos\left(a-\frac{\pi}{2}\right) = -\cos\left(a+\frac{\pi}{2}\right)$



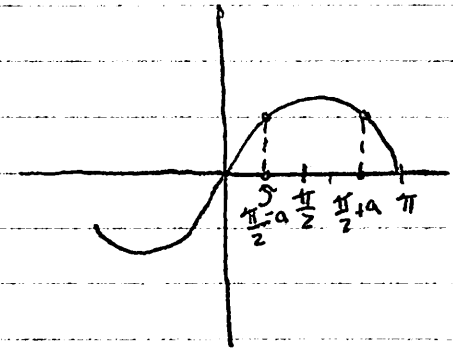
⑨



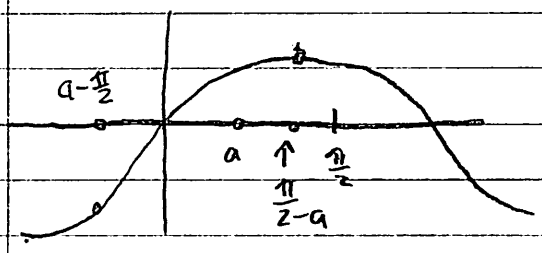
⑪ $\sin(\pi-a) = \sin A$



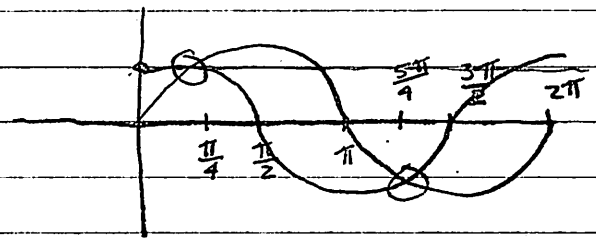
⑬ $\sin\frac{\pi}{2}+a = \sin\frac{\pi}{2}-a$



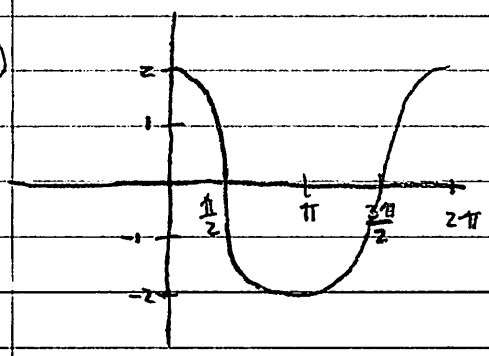
(15) $\sin(a - \frac{\pi}{2}) = -\sin(\frac{\pi}{2} - a)$



(17)



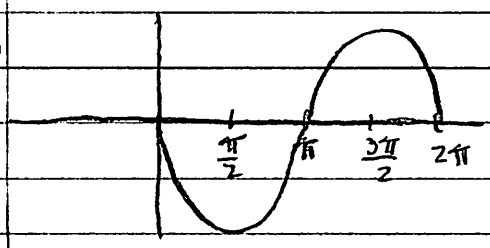
(21)



$y = 2\cos x$

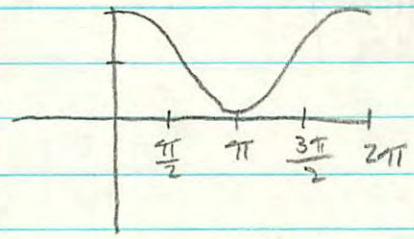
doubles the amplitude in distance between max + min of fctn.

(23)



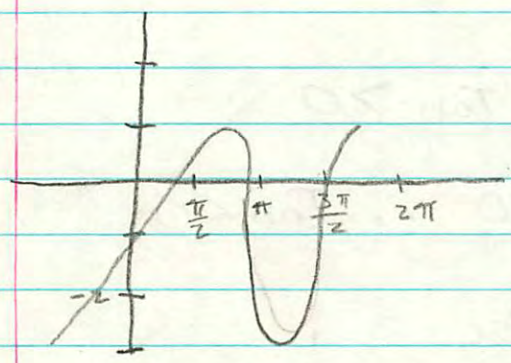
$y = -\sin x$

(25)



$y = \cos x + 1$
raised 1 unit

(27)



$y = 2\sin x - 1$

double amplitude and graph is lowered 1 unit

31) $y = a \cos x \quad a > 0$ amplitude $= a$

32) $y = a \cos x \quad a < 0$ amplitude $= a$ graph upsidown

33) $y = \cos x + b \quad b > 0$ raised b units

34) $y = \cos x + b \quad b < 0$ lowered b units

35) $y = a \cos x + b$ a = amplitude raised b units
$a > 0 \quad b > 0$

36) $y = a \cos x + b$ a = amplitude raised b units
$a < 0 \quad b > 0$ + upsidedown.

37) $f(x) = \cos 2x$
$f(x + \pi) = \cos 2(x + \pi)$
$= \cos 2x + 2\pi$
$= \cos 2x$
$= f(x)$

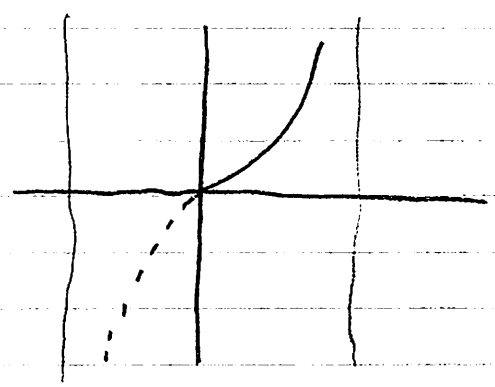
38) $g(x) = \sin \frac{1}{2} x$
$g(x + 4\pi) = \sin \frac{1}{2}(x + 4\pi)$
$= \sin \frac{1}{2} x + 2\pi$
$= \sin \frac{1}{2} x$
$= g(x)$

4th quad	1st quad	2nd quad	3rd quad	4th quad
	(0°) $\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	

In $\underline{1}^{st}$ Quad $S+C>0 \therefore \tan>0$

In 4th Quad $S<0$ $C>0$ $\therefore \tan<0$

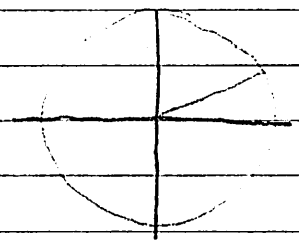
Same Style for Sec + Csc.



3-2
Pg 102-106

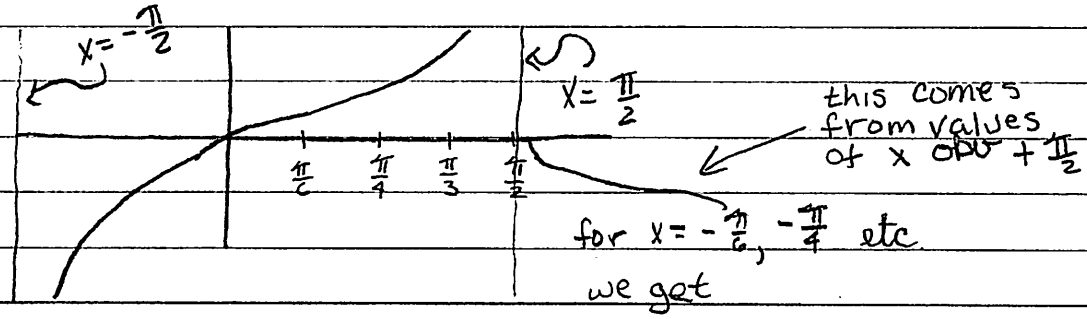
Graph of tan, sec, csc + cot.

Since $\tan x = \frac{\sin x}{\cos x}$ we can make a table of values



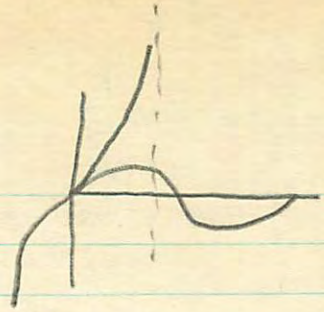
x	sin	cos	tan
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	∞

From these graph tan x



This tan has period π rather than 2π like Sin + Cos. So we know the next period will begin

#20



$|\tan x| \geq |\sin x|$

for $0 \leq x < \frac{\pi}{2}$

$\tan x > 0 \quad \sin x > 0$

$\therefore$

$|\tan x| \geq |\sin x|$

$\tan x \geq \sin x$

$\sin x \geq \sin x \cos x$

$1 \geq \cos x$

$\frac{\pi}{2} < 0 \leq \pi$

$\tan x = \frac{\sin x}{\cos x}$ where $\cos x < 0$

$\sin x > 0$

$-\tan x \geq \sin x$

$\frac{\sin x}{-\cos x} \geq \sin x \quad (-\cos x > 0)$

$\sin x \geq -\cos x \sin x$

$-1 \leq \cos x$

$\pi \leq x < \frac{3\pi}{2}$

$\tan x > 0 \quad \sin x < 0$

$\frac{\sin x}{\cos x} \geq -\sin x$

$\sin x \geq -\sin x \cos x$

$-1 \leq \cos x$

$\frac{3\pi}{2} \leq x \leq 2\pi$

$\tan x < 0 \quad \sin x < 0$

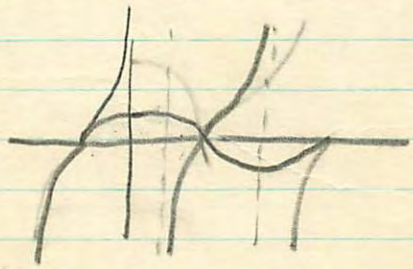
$\frac{-\sin x}{\cos x} \geq -\sin x$

$-\sin x \geq -\sin x \cos x$

$\frac{-\sin x}{-\sin x} \geq \cos x$

$1 \geq \cos x$

#21



$0 < x \leq \frac{\pi}{2}$

$\cot x \geq \cos x$

$\cos x \geq \cos x \sin x$

$1 \geq \sin x$

$\frac{\pi}{2} \leq x < \pi$

$\frac{-\cos x}{\sin x} \geq -\cos x$

Note $-\cos x > 0$ $\therefore$ no change required

$-\cos x \geq -\cos x \sin x$

$1 \geq \sin x$

$\pi < x \leq \frac{3\pi}{2}$

$\frac{-\cos x}{-\sin x} \geq -\cos x$

$-\cos x \leq +\cos x \sin x$

$-1 \leq \sin x$

$(\sin x < 0)$

$\frac{\cos x}{-\sin x} \geq \cos x \Rightarrow \cos x \leq -\cos x \sin x$

$-1 \geq \sin x$

$\frac{-\cos x}{\sin x} \geq \cos x \Rightarrow -\cos x \geq \cos x \sin x$

$-1 \geq \sin x$

Since tan is not defined at $\frac{\pi}{2}$, it is understandable that our graph has no value at $\frac{\pi}{2}$ or $\frac{K\pi}{2}$ where K-odd These are called "vertical asymptotes" of the fctn.

HW Pg 105 17-19

3-3 In the first HW. of graphing Sin + Cos the constants caused much problems. Just a passing Note about the general rules concerning those constants. See Pg. 106 figure 3-20. This shows the change caused by $\underline{y = a\cos x}$ a then is the amplitude.

for $y = \cos x + a$ we have only a shift up or down of the graph. and finally for $y = \cos ax$. The "a" here changes the period.

This is Summerized on Pg 109 using

$y = a\cos(bx + c) + d$

a => amplitude

b => Period the period of the fctn is $\frac{2\pi}{|b|}$

$\frac{c}{b}$ => Phase shift is left or right movement.

d => up or down location of graph

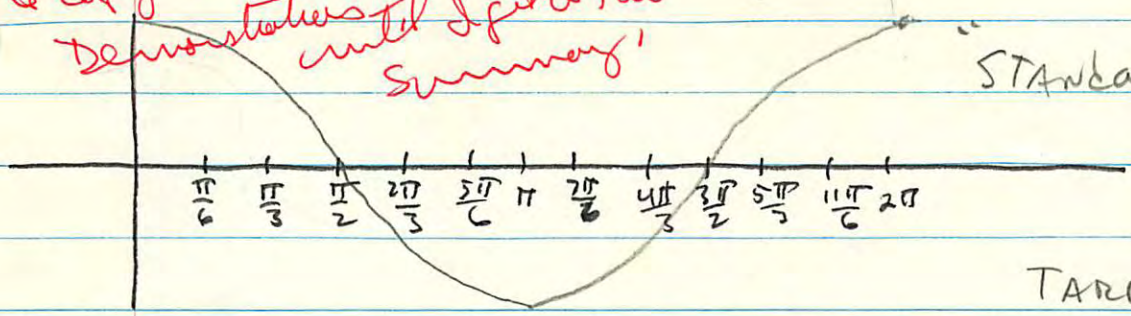
It is intended that you should sketch not plot these graphs

Tell Students to watch not to copy this Demonstration until I get to the summary'

$y = a\cos(bx - c) + D$

$y = a\cos b\left(x - \frac{c}{b}\right) + D$

"Squeeze Then Shift" lecture.



"STANDARD" COSINE

TARGET fctn is

$y = \cos(3x - \frac{3\pi}{2})$

$y = \cos x$

Notice $\frac{1}{4}$th of the period it is at zero and $\frac{\text{period}}{4}$

The 2nd fourth it is at its lowest point

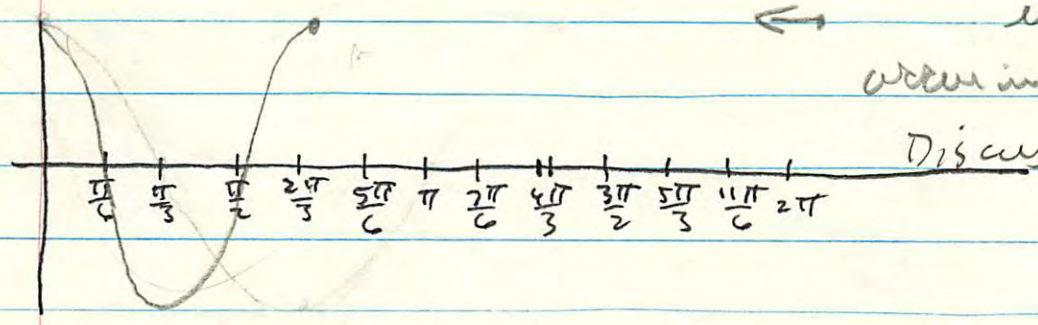
The 3rd Quarter it is at zero again

the last Quart ends at its max again,

$y = \cos 3(x - \frac{\pi}{2})$

Since old period is 2π then new period is $\frac{2\pi}{3}$ ie entire fctn will occur in a total length of $\frac{2\pi}{3}$.

Discuss $\frac{1}{2}$ way through. etc



we always Squeeze to $x=0$.

Hold $(0,1)$ Fixed and Squeeze $(2\pi, 1)$ Back to $(\frac{2\pi}{3}, 1)$.

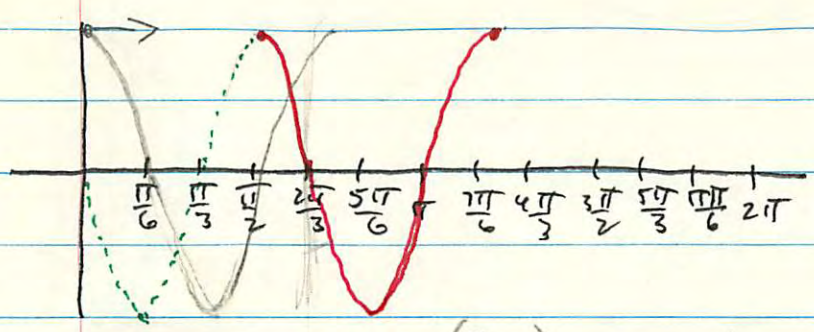
Now the curve has a total length of $\frac{2\pi}{3}$, ie its period is $\frac{2\pi}{3}$

the equation for this "New" curve is $y = \cos 3x$.

Notice $\frac{1}{4}$ through period $= \frac{1}{4} \cdot \frac{2\pi}{3} = \frac{\pi}{6}$, $\frac{1}{2} \cdot \frac{2\pi}{3} = \frac{\pi}{3}$, etc.

The "b" in the General Formula defines the "Period".

"New Period" $= \frac{\text{Normal Period}}{b} \Rightarrow \frac{2\pi}{3}$



Move $(0,1)$ to $\left(\frac{\pi}{2}, 1\right)$. we have "shifted" the curve $\frac{\pi}{2}$ to the Right.

Now what is the equation of the "Red" curve?

we "back up" the curve.

Going Back $\frac{1}{4}$ of Period yields $\left(\frac{\pi}{3}, 0\right)$

Going Back another Quarter yields $\left(\frac{\pi}{6}, -1\right)$ etc.

the Green curve looks like a $-\sin$ curve because it "starts" at $(0,0)$ and goes down.

Our Red curve is $Y = \cos 3\left(x - \left(\frac{\pi}{2}\right)\right)$

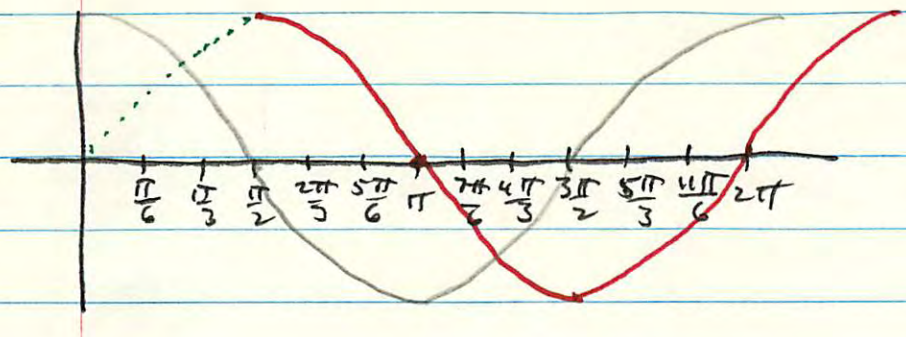
Period is still $\frac{2\pi}{3}$ $\quad$ $\frac{\pi}{2}$ shift to right.

$$Y = \cos\left(3x - \frac{3\pi}{2}\right)$$

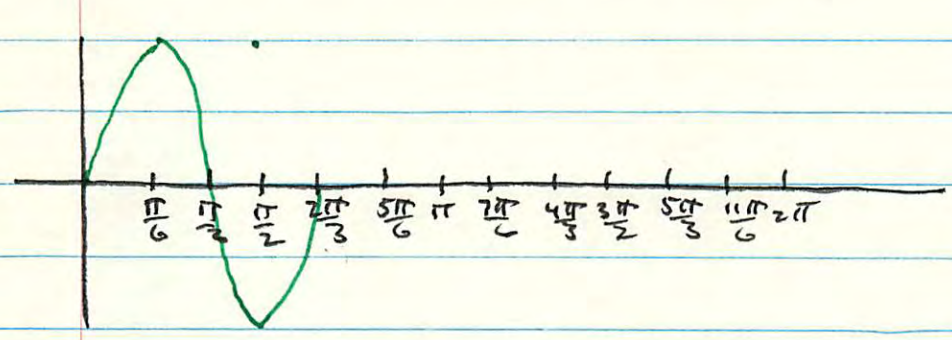
$$\cos 3x \underbrace{\cos\frac{3\pi}{2}}_{0} + \sin 3x \underbrace{\sin\frac{3\pi}{2}}_{-1}$$

$$= -\sin 3x$$

we have confirmed that our graph is correct.



Start again with $y = \cos x$
Shift everything $\frac{\pi}{2}$ to the right. (the red curve)
"Back the curve up to zero"



this is the curve $y = \sin 3x$.
Clearly it is the wrong graph. why did we end up with the wrong graph?
This second version involved first a shift then a squeeze.
The rule is Charleton Heston says
"1st Squeeze to Zero
Student then Shift."

Rebel w/o a cause.

Notes begins Now
Now Show "using Zeros" method for Cosine & Sine

$\sec\left(\frac{1}{2}x + \frac{\pi}{12}\right)$

$\sec \frac{1}{2}\left(x + \frac{\pi}{6}\right)$

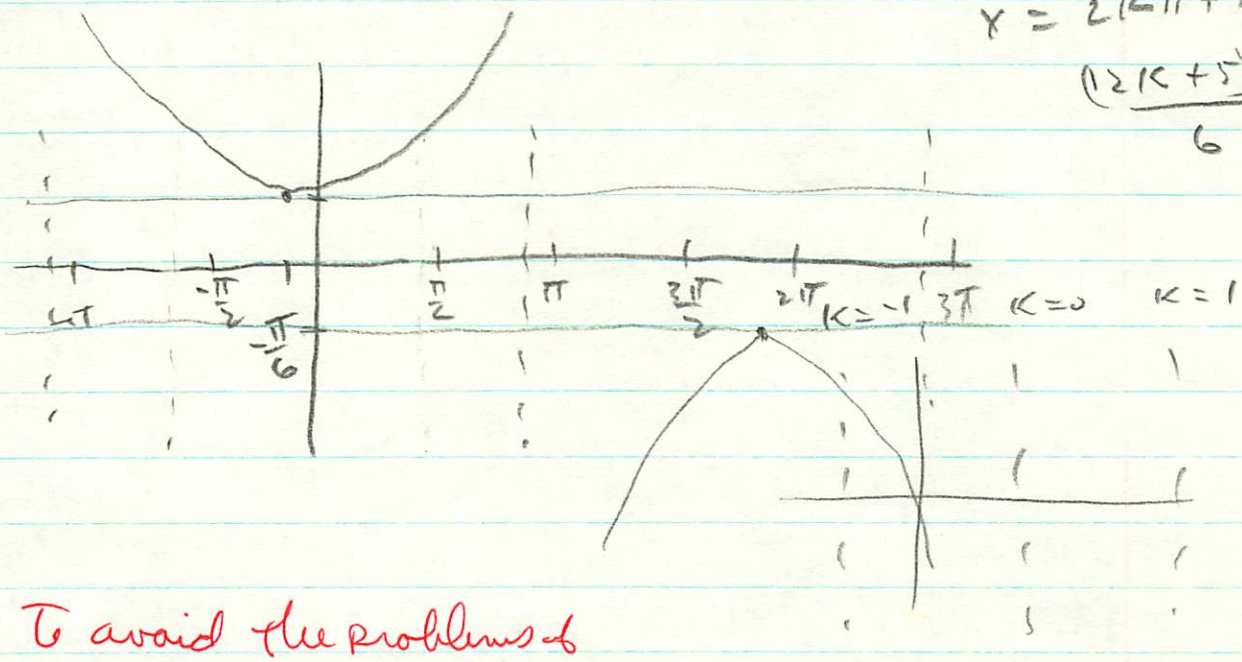
Per 4π

$\frac{1}{2}\left(x + \frac{\pi}{6}\right) = \frac{(2k+1)\pi}{2}$

$x + \frac{\pi}{6} = 2k\pi + \pi$

$x = 2k\pi + \frac{5\pi}{6}$

$\frac{(12k+5)\pi}{6}$



To avoid the problems of squeeze then shift,

use asymptotes method.

this even works on sin & cos

if you consider "Axis zeros" as asymptote

$k=-1$ $\quad -\frac{7\pi}{6}$ $\quad \frac{12\pi}{6}$

$k=0$ $\quad \frac{5\pi}{6}$ $\quad 2\pi$ $\quad \frac{11\pi}{6}$

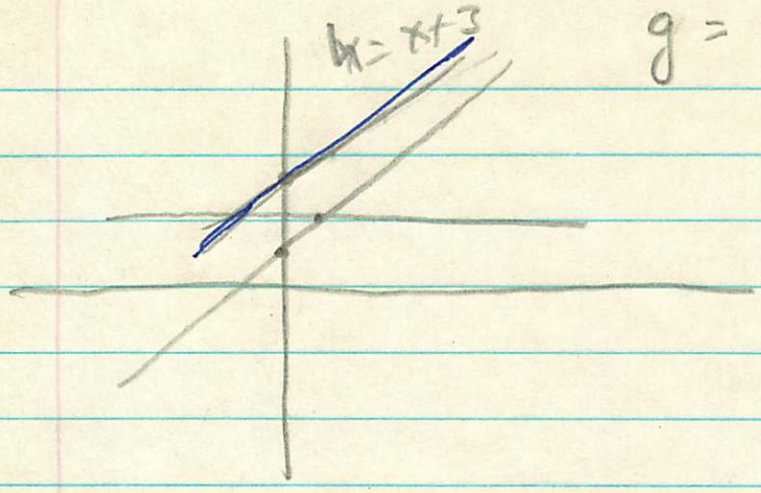
$k=1$ $\quad \frac{17\pi}{6}$

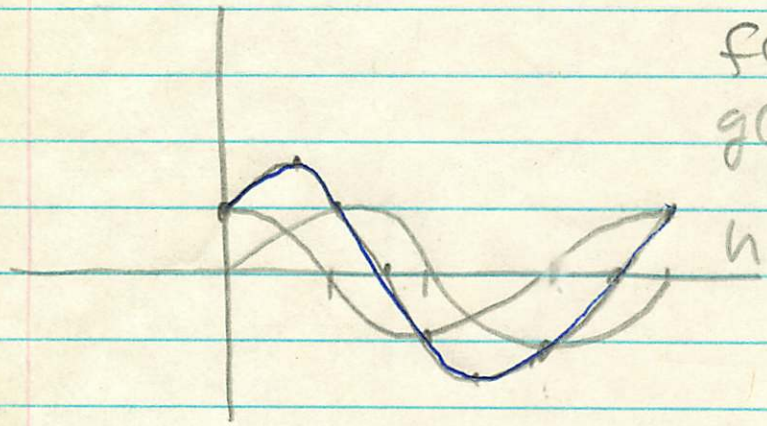
Ch 3
~~Silent lecture~~

$f = x + 1$

$g = 2$

teach use of TI 82

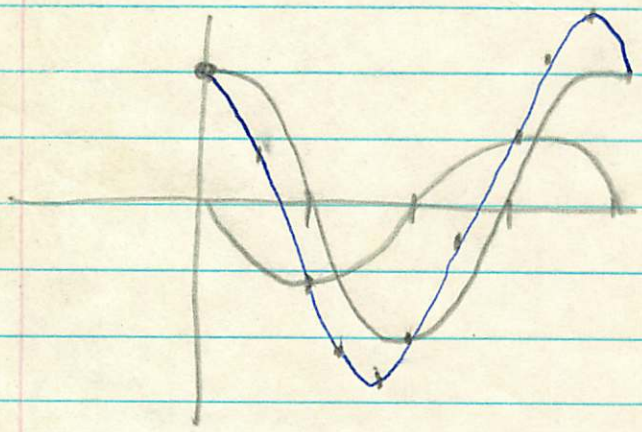




$f(x) = \cos x$

$g(x) = \sin x$

$h(x) = \sin x + \cos x$



$f(x) = 2\cos x$

$g(x) = -\sin x$

$h(x) = 2\cos x - \sin x$

$y = a\cos(bx+c)+d$

#1

a $\quad a>0 \begin{cases} 0<a<1 & \text{decreases} \updownarrow \text{ of Graph arm cox} \\ 1<a & \text{increases} \updownarrow \text{ of graph} \end{cases}$

$a=0 \Rightarrow x$ axis

$a<0 \begin{cases} -1<a<0 & \text{reflect about xaxis Decreases} \\ a<-1 & \text{reflect about xaxis increases.} \end{cases}$

d $\quad d>0$ Graph Raised

$d=0$ Graph is std position

$d<0$ Graph Lowered.

think $a\cos(bx+c)=0$ then $y=d$

b) $\quad b>0 \begin{cases} b>1 & \text{Period length decreased} \\ 0<b<1 & \text{Period length increased} \end{cases} \Big\} \frac{2\pi}{|b|}$

$b=0$ Depends on Trig function

$b<0 \left\{ \text{use applicable identity to convert to } b>0. \right.$

c) Phase shift by $\frac{c}{b}$

$\frac{c}{b}<0$ Graph moved to Left

$\frac{c}{b}=0$ No Phase shift

$\frac{c}{b}>0$ Graph shifted Right.

END of Lecture

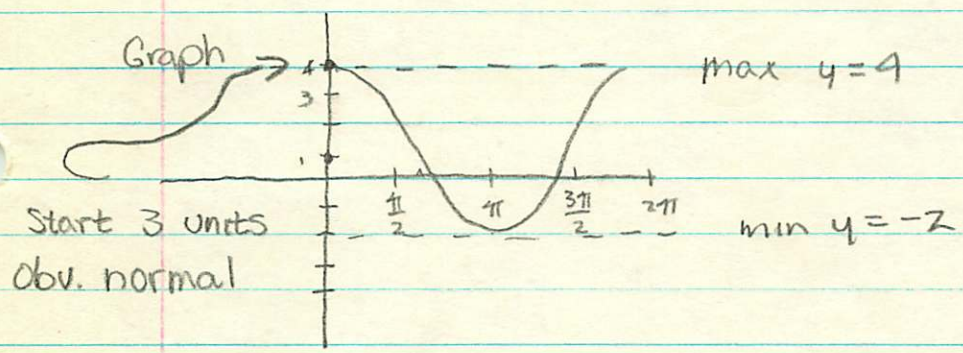
Pg 109 ex. of the sketch

$a=3$ which means that amplt. will be 3
if $a<0$ Amplitude is still a but graph is turned over.

$b=1$ which means period is 2π

$c=0$ Thus No right or left change.

$d=1$ graph is raised 1 unit.



$y = 3\cos x + 1$

The overall idea of the sketch is a quick method of seeing just what a fctn will do. It reminds you of max or min values where the fctn is not defined etc.

HW 3-2 + 3-3

Pg 105

(17) $x = K\pi$, $K \in J$

(18) $x = (2K+1)\frac{\pi}{2}$ $K \in J$

(19) $x = K\pi$ $K \in J$

(21.) $\frac{\sin}{\cos} \geq \sin$

$\frac{1}{\cos} \geq 1$

$\sec \geq 1$

which is range of sec. ∴ original line in true.

(22) Same style

Pg 109

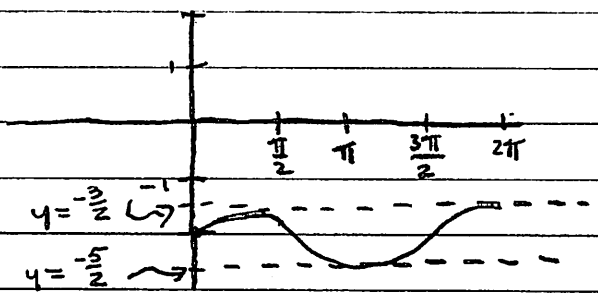
(2) $y = \frac{1}{2}\sin x - 2$

amp $= \frac{1}{2}$

Period $= 2\pi$ $(b=1)$

Phase shift $= 0$ $(c=0)$

Location = down 2



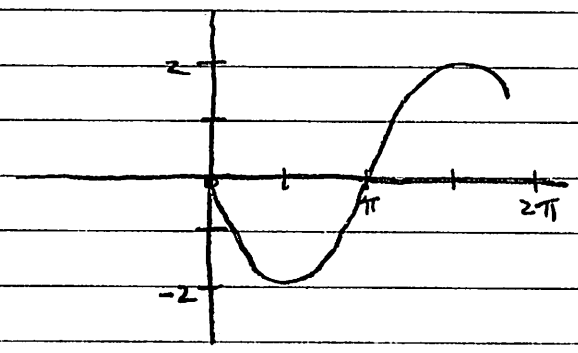
(3) $y = -2\sin x$

amp = 2 Graph upsidedown

period 2π

Phase Shift 0

Location - normal



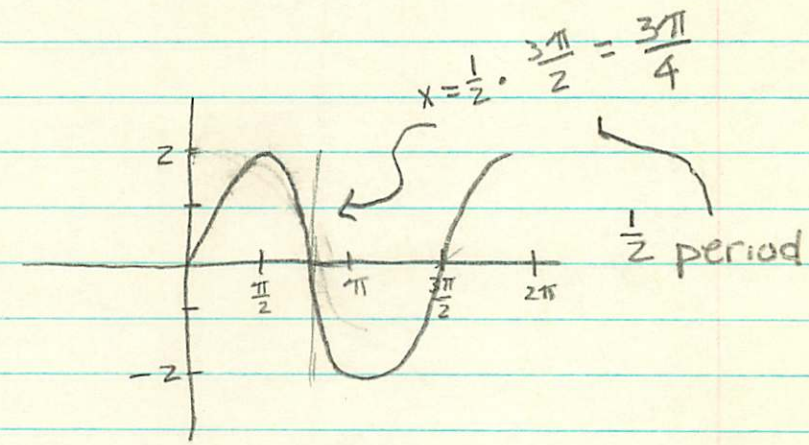
(5) $y = 2\sin 3x$

amp 2

period $\frac{3}{2}$

shift 0

Location - normal



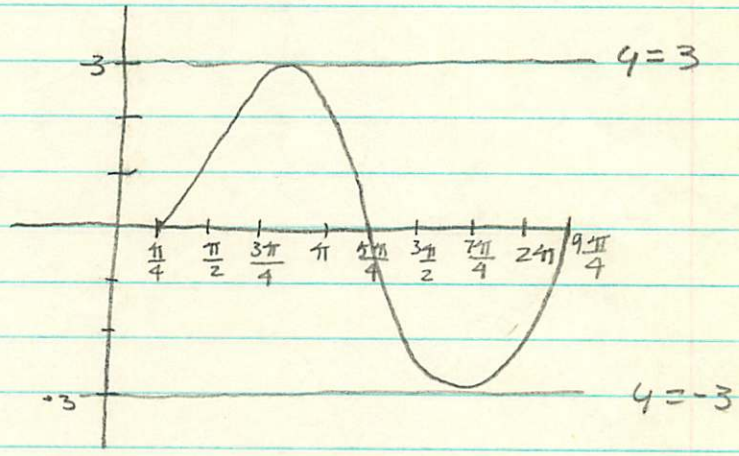
(8) $y = 3\sin\left(x - \frac{\pi}{4}\right)$

amp 3

period 2π

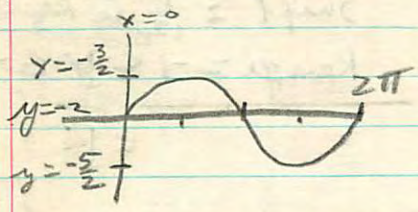
shift to right $\frac{\pi}{4}$

Location normal



$y = a\cos(bx+c)+d$ Period $= \frac{2\pi}{b}$ because cos has period 2π.

#2 $y = \frac{1}{2}\sin x - 2$

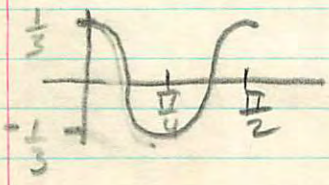


Amp $= \frac{1}{2}$
Period $= 2\pi$
Phase shift = None
Range $\Rightarrow -\frac{3}{2} \geq y \geq -\frac{5}{2}$

#4 $y = -3\cos x$

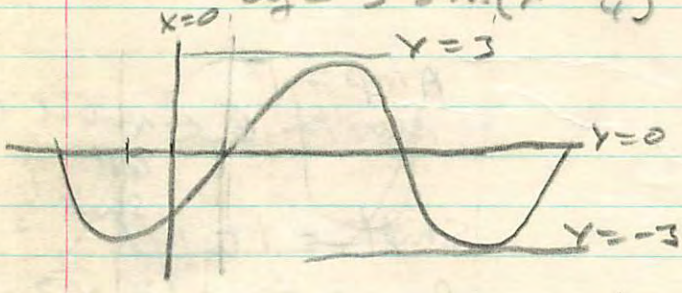
Amp $= 3$
Period $= 2\pi$
Phase = None
Range $\Rightarrow 3 \leq y \leq -3$

#6 $y = \frac{1}{3}\cos 4x$



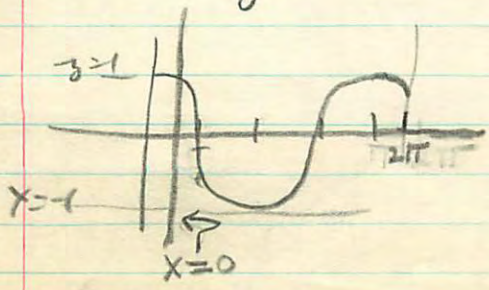
Amp $= \frac{1}{3}$
Period $= \frac{\pi}{2}$
Phase = None
Range $= \frac{1}{3} \leq y \leq -\frac{1}{3}$

#8 $y = 3\sin(x - \frac{\pi}{4})$



Amp $= \frac{1}{3}$
Period $= 2\pi$
Phase shift $= \frac{\pi}{4}$ to right.
Range $= 3 \leq y \leq -3$

#10 $y = \cos(2x + \frac{\pi}{4})$
$y = \cos 2(x + \frac{\pi}{8})$

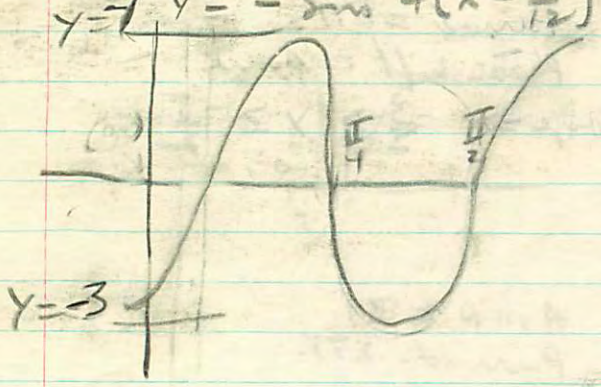


ampl $= 1$
Period $= \pi$
Shift $= \frac{\pi}{8}$ to left
Range $\Rightarrow -1 \leq y \leq 1$

110

#12 $y = -\sin\left(4x - \frac{\pi}{3}\right) - 2$

$y = -\sin 4\left(x - \frac{\pi}{12}\right) - 2$

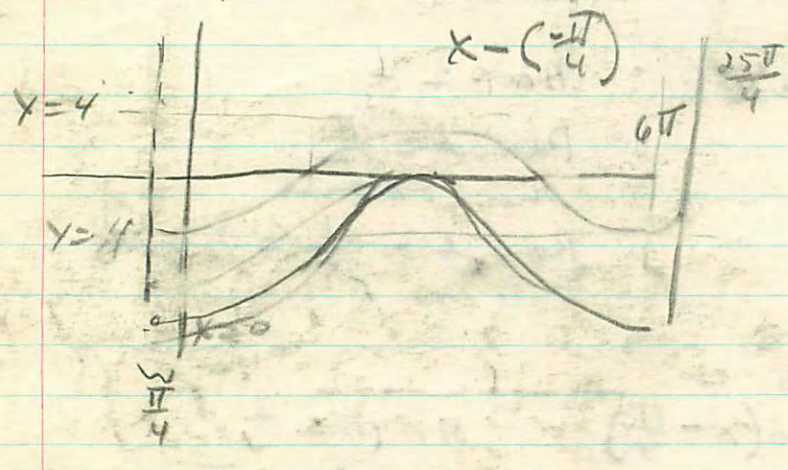


Amp = 1
Period = $\frac{\pi}{2}$
Shift = $\frac{\pi}{12}$ to right.
Range = $-1 \le y \le -3$

#14 $y = -2\cos\left(\frac{1}{3}x + \frac{\pi}{12}\right) - 2$

$y = -2\cos\frac{1}{3}\left(x + \frac{\pi}{4}\right) - 2$

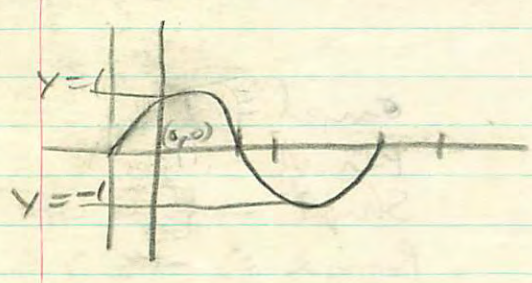
$x - \left(\frac{-\pi}{4}\right)$



Amp = 2
Period = 6π
Shift = $\frac{\pi}{4}$ to left.
Range = $+4 \ge y \ge -4$

#16 $y = \sin\left(\frac{\pi x}{3} + \frac{\pi}{3}\right)$

$y = \sin\frac{\pi}{3}(x+1)$

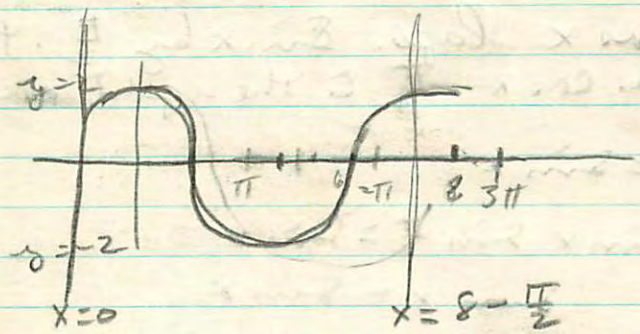


Amp = 1
Period = 6 ← noted not 6π but simply 6.
Shift = 1 to left.
Range ⇒ $1 \ge y \ge -1$

18 $y = 2\cos\left(\frac{\pi y}{4} - \frac{\pi}{8}\right)$

$y = 2\cos\frac{\pi}{4}\left(x - \frac{\pi}{2}\right)$

amp $= 2$
Period $= 8$
Shift $= \frac{\pi}{2}$ Right
Range $= 2 \le y \le -2$



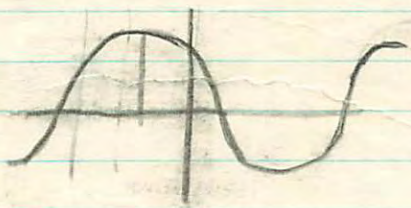
$x - \left(\frac{\pi}{2}\right)$

#19 $y = -\cos x$

WTP $-\cos x = \cos(x+\pi)$
$= \cos x \cos\pi - \sin x \sin\pi$
$= -\cos x$

#20 WTP we are Given $\cos(x+\frac{\pi}{4})$ (#8 #1). we want to show that if we shift $\cos(x+\frac{\pi}{4})$ $\frac{\pi}{4}$ more to the left we will have $-\sin x$
Thus $\cos(x+\frac{\pi}{4}+\frac{\pi}{4}) = -\sin x \Rightarrow$

#1



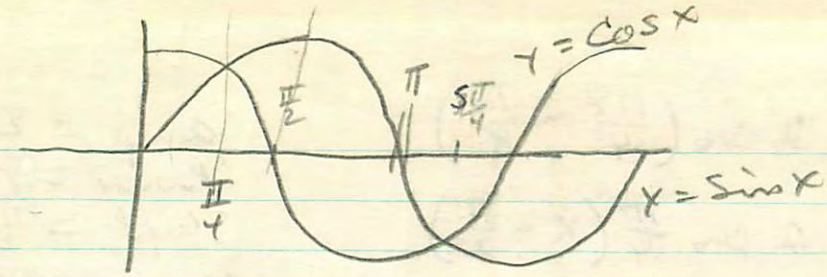
$\cos(x+\frac{\pi}{2}) = -\sin x$

$\cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = -\sin x$

$-\sin x = -\sin x$

110

21



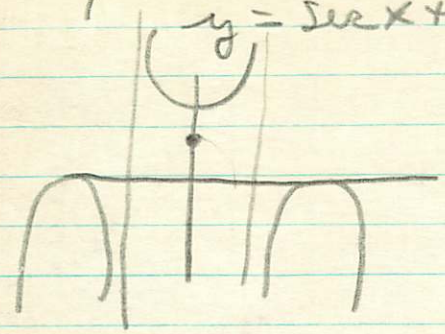
we see that Cos x lags Sin x by $\frac{\pi}{2}$. that is we need to move Cos x $\frac{\pi}{2}$ to the right or

$$\cos\left(x - \frac{\pi}{2}\right) = \sin x$$

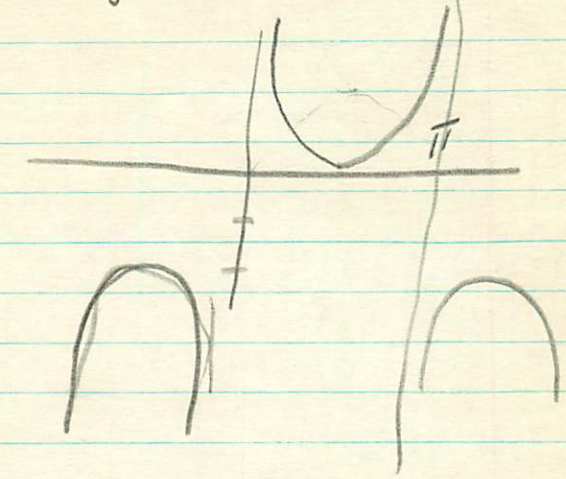
$$\text{as } x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2} = \sin x$$

$$\sin x = \sin x$$

1 $y = \sec x + 1$

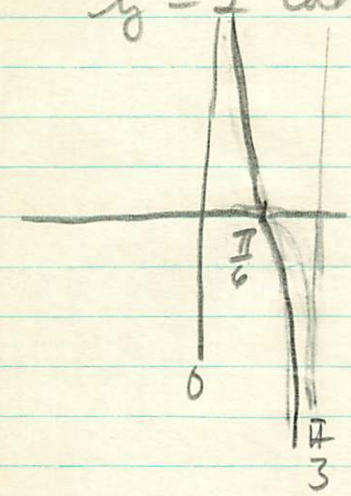


2 $y = \csc x - 1$



3 $y = 2 \cot 3x$ Period $\frac{\pi}{3}$

Stretched, ie amplitud doubled.



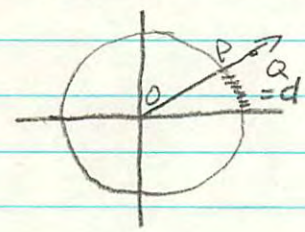
(4) $y = 2 \csc 2x$

3-7 Uniform Motion,

Given a unit circle we have a ray $\overrightarrow{OQ}$ from the center. If this ray moves around the circle at a constant speed or if the circle turns under the fixed ray, we have what we call uniform circular motion.

Everybody remembers the relationship between Rate, time, and distance. ie. $RT = D$

Here the distance is our arc length



It is said then that gives a speed or velocity, the arc length depends on or is a function of time.

Let T be time required to complete one revolution, or in terms of arc length 2π. From $R \cdot T = D$, $\omega \cdot T = 2\pi$ we have $R = \frac{D}{T}$ or $\frac{2\pi}{T} = R$ In circular problems we use (ω) to represent this rate (rotational velocity) while r = rate or linear velocity. distance thus;

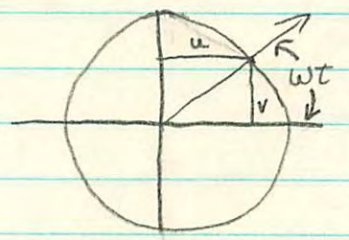
one item to remember. $\omega = \frac{2\pi}{T}$

I am working with a unit circle, thus the circumference is 2π. The general formula for circumference is $c = 2\pi r$ but $r = 1$

$\omega = \frac{2\pi}{T}$ <u>Notice</u> T is the time needed for <u>ONE</u> revolution.

If we want to know how far P moves on the arc length, we have the rate ω times the time t (small t) or $(\frac{2\pi}{T})t$ = arc length traveled by P in t units of time.

The coordinates are the next problem. What are the co-ord. of a point on the circle? Using (u, v) instead of (x, y)



Because of the confusion of too many x's we find the co-ord's to be $(\cos \omega t, \sin \omega t)$. (see pg 48 figure 2-6 for descriptions of arc length).

This is only true on our unit circle! On a circle of radius a we have $(a \cos \omega t, a \sin \omega t)$.

Notice also in diagram we started at the point $(1,0)$ at time $t=0$. If we start at a pt. of arc length b from $(1,0)$ we have:

$$(\cos \omega t + b, \sin \omega t + b)$$

as ending co-ord. and beginning coord. of.

$$(\cos b, \sin b).$$

Ex 1 Page 121

Start at (1,0)

$\frac{4}{3}$ revolutions per Sec. ie.

$\frac{4}{3}$ Rev. = 1 sec. $\Rightarrow$ 1 Rev. = $\frac{3}{4}$ sec.

Our speed or rotational velocity of the ray $\overrightarrow{OQ}$ is $\omega = \frac{2\pi}{T}$ not necessarily that if P as in later examples.

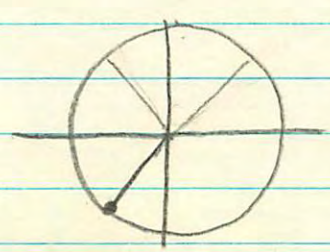
$$\omega = \frac{2\pi}{\frac{3}{4}} = \frac{8\pi}{3}$$

Coordinates of P after 2 seconds using:

$u = \cos(\omega t + b)$ and $b = 0$ we have
$v = \sin(\omega t + b)$

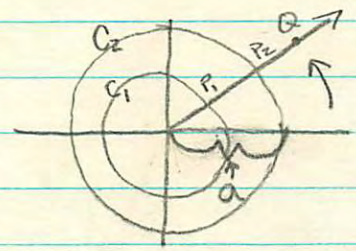
$u = \cos\left(\frac{8\pi t}{3}\right)$ but t is 2 sec.
$v = \sin\left(\frac{8\pi t}{3}\right)$ thus

$$u = \cos\frac{16\pi}{3} \Rightarrow \cos\left(4\pi + \frac{4\pi}{3}\right) = \cos\frac{4\pi}{3} = -\frac{1}{2}$$
$$v = \sin\frac{16\pi}{3} \Rightarrow \sin\left(4\pi + \frac{4\pi}{3}\right) = \sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$$



Following along in the book (pg. 122)

If we have the same basic set up ie. our unit circle but with another circle with radius a.



P_2 it is clear will travel farther than P_1. P_1 travels the circle at a rate of $\omega = \frac{2\pi}{T}$. P_2 travels farther in the same time thus its speed is $a\omega = \frac{2a\pi}{T}$ (where a is distance from center). ($\frac{2\pi}{T} = \omega$ multiplied by a).

It should be remembered that usually we are talking about our unit circle, thus the circumference is 2π. For a circle with radius 3 the circumference is 18π ($2 \cdot 3^2\pi$).

One other item to be mentioned is the fact that our ray has rotated counterclockwise. This is said to be in a + or positive direction. Clockwise is − or Negative.

The negative generally only shows directions, the speed itself is still + and the time is $|t|$ (See ex. 14)

Ex 2. Pg 123

a) what is ω? (Rotational Velocity).

Given $33\frac{1}{3}$ RPM => $\frac{100}{3}$ RPM => $\frac{3}{100}$ = 1 rev.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{3}{100}} = \frac{200\pi}{3}$

(If Questions of "any ray" comes up....

Our ray will trace a circle 1" from center.
Since we are using inches ie. 5" record.

b) At 3 inches from center how fast?

3 of our units or $3x \cdot \frac{200\pi}{3} = 200\pi$

Do ex 12. for class

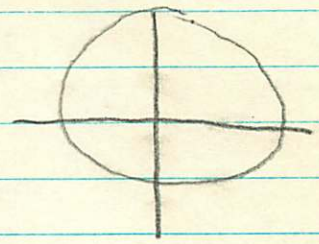
HW. 1-7 all

10, 11, 13, 14

ω is <u>rotational</u> velocity

$A\omega$ is <u>Linear</u> velocity!

Hw Pg 123 —



① $\omega = \frac{2\pi}{3}$ $r = 3$ $t = 4$

$RWt = \frac{2\pi}{3} \cdot 3 \cdot 4 = 8\pi$

② $\omega = \frac{5\pi}{12}$, $r = 2$ $t = 6 \Rightarrow 5\pi$

③ $\omega = \frac{4\pi}{3}$ $r = 6$ $t = \frac{5}{4}$ $\frac{4}{3} \cdot \frac{5}{4} \cdot \frac{2}{6} \cdot \pi = 10\pi$

④ $\omega = \frac{\pi}{3}$ $r = 10$ $t = \frac{9}{2}$ $\frac{\pi}{3} \cdot \frac{5}{10} \cdot \frac{3}{9}{2} = 15\pi$

⑤ $\frac{3\pi}{4} \cdot \frac{5}{2} \cdot 3 = \frac{45\pi}{8}$

⑥ $\frac{5\pi}{4} \cdot \frac{2}{3} \cdot \frac{4}{16}{3} = \frac{40\pi}{9}$

⑦ a) $\omega = \frac{\frac{2\pi}{3}}{\frac{3}{4}} = \frac{8\pi}{3}$ 4 rev = 3 sec $\frac{8\pi}{3}$ sec.

1 rev = $\frac{3}{4}$ sec

b) radius 3in $\Rightarrow \frac{8\pi}{3}$ sec 3in = 8π in/sec.

⑧ 1 rev = 60 min

ⓑ $\omega = \frac{2\pi}{60} = \frac{\pi}{30}$ min $\Leftarrow$ unit

$\frac{\pi}{30} \cdot 4 \cdot \frac{2\pi}{15}$

4 inches long $\Rightarrow \frac{4\pi}{30} = 4\omega \equiv 4''$

time 25 min $\Rightarrow \frac{25 \cdot 4\pi}{30 \cdot 3} = \frac{10\pi}{3}$

ωt

What happened to the Neg. Velocity? $\Rightarrow$

Absolute value.

The distance is the same, the negative only means is opposite direction. The only time the negative would make a difference would be to locate coordinates.

(9) 160 rev. = 1 min

$1 \text{ rev} = \frac{1}{160} \text{ min}$

$\omega = \frac{2\pi}{\frac{1}{160}} = 320\pi/\text{min}$

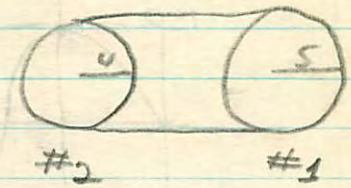
it rolls the same distance as it's circumference

$\therefore 3 \cdot 4 \cdot 320\pi = 3840\pi/\text{min}$

this is radius

#10 in b Part exchange the functions of the two wheels. ie 8" wheel is driving wheel.

a) (original problem)



10" wheel drives.

$168 \text{ RPM} \Rightarrow \omega_5 = \frac{2\pi}{\frac{1}{168}} = 336\pi$

Speed of belt $= a\omega$, ie a point on wheel #1 moves 3 inches then the belt also moves 3".

$336\pi \cdot 5 = 5\omega = 1680\pi$

Since $5\omega_5 = 4\omega_4$ ie speed of belt equals speed of 8" wheel at the rim.

Thus $\omega_4 = \frac{5}{4}\omega_5 \Rightarrow \frac{1680\pi}{4} = 420\pi$

b) Reversed roles

Driving wheel (8") rotates at $168 \text{ RPM} \Rightarrow \omega_4 = \frac{2\pi}{\frac{1}{168}} = 336\pi$

Linear velocity then of 8" wheel $= 4 \cdot 336\pi =$

$1344\pi = 4\omega_4$

for same reason as above. $5\omega_5 = 1344\pi \Rightarrow$

$\omega_5 = \frac{1344\pi}{5} \approx 269\pi$

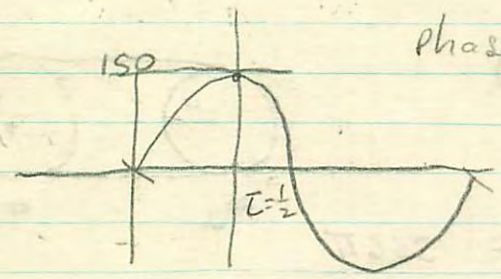
$E = E_{max} \sin(\omega t + b), \quad -\frac{\pi}{2} \le b \le \frac{\pi}{2}.$ $\qquad \omega = 120\pi.$

The answer in the text is wrong. $\qquad$ When $t = \frac{1}{2}$, E is max, i.e. $E = E_{max} = 150$

If b were 0, then $E = 150 \sin(60\pi + 0) = (150)(0) = 0$.

not max!

phase shift $\frac{\pi}{2}$



$y = a \sin \omega t \qquad \omega = 1$ period $T = 2\pi \Rightarrow \omega = 1$

$\omega = \frac{2\pi}{T} = 1$

B

4 sin

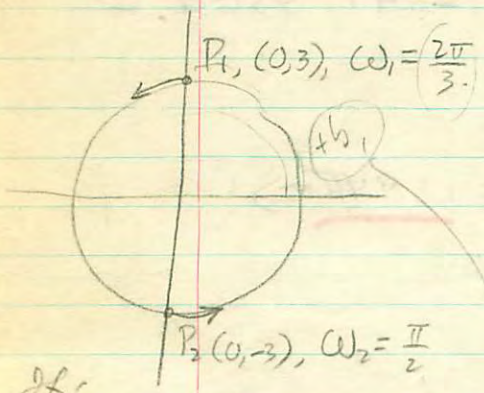
graph:

$y = \sin\left(\frac{\pi x}{2} + \pi\right)$

$y = \sin \frac{\pi}{2}(x + 2)$ $\qquad$ T = time for one rev.

phase shift $\qquad$ Period $= 4$.

A test prob to be used in the next test $\qquad$ Good problem



① Find t when P_1, P_2 meet.

② How far have P_1, P_2 travel

③ Find the coord.

It's not unit circle

Phase shift.

$3 : \frac{2\pi}{3}$

$D_{P_1} = D_{P_2} + 3\pi$

$3\cos\left(\frac{2\pi}{3}t + b_1\right), \; 3\sin\left(\frac{\pi}{2}t + b_2\right)$

$3 \cdot \frac{2\pi}{3} \cdot t = \frac{3\pi}{2} t + 3\pi$

$2\pi t = \frac{3}{2} t + 3$

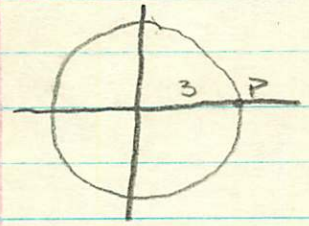
$2t = \frac{3}{2} t + 3$

$\frac{1}{2} t = 6$

$t = 6$

to justify $t<0$

(11)



$t=0$ $\quad \omega = \frac{4\pi}{3}$

$\frac{4\pi}{3} \cdot 3''$

Clock running now when stop watch is started we have $t=0$

$u = \cos 4\pi t \qquad t = \frac{3}{2}$

$v = \sin 4\pi t$

$(u, v) = (\cos 6\pi, \sin 6\pi) = (0, 3)$ (a)

(Neg direction see problem 8) $\quad (0, 3)$ (b)

Notice that ρ does not end up ~~inside~~ ~~of~~ as part of the angle.

i.e. $(\rho \cos \omega t + b, \rho \sin(\omega t + b))$

where $\cos \rho \omega t$ etc.

The angle is always computed on the unit circle.
(which is what ωt does)

Look

#12

Velocity $P_1 = \frac{4\pi}{3}$ Velocity $P_2 = \frac{\pi}{2}$

Distance traveled is given by $R\omega\tau$.
where R is radius, ω is rotational velocity and τ is time traveled.

$3 \cdot \frac{4\pi}{3} \cdot \tau = D_1$ $\qquad$ $3 \cdot \frac{\pi}{2}\tau = D_2$

$4\pi\tau = D_1$ Yes $\qquad$ $\frac{3\pi}{2}\tau = D_2$ Yes.

Since P_1 has made 1 more revolution than P_2 P_1 traveled an additional $3 \cdot 2\pi$ $(2\pi R)$ amount. ∴ to make the equations balance we add 6π to P_2

$$\frac{4\pi\tau}{3} = \frac{3\pi}{2}\tau + 6\pi$$

$$10\pi\tau = 6\pi$$

$$5\pi\tau = 12\pi$$

$$\tau = \frac{12}{5} \text{ Yes}$$

we know $(u_1, v_1) = (u_2, v_2)$ ∴

what are the coordinates at $\tau = \frac{12}{5}$?

$(u, v) \Rightarrow (a\cos\omega\tau, a\sin\omega\tau) \Rightarrow$

$\left(3\cos\frac{4\pi}{3}\tau, 3\sin\frac{4\pi}{3}\tau\right) \Rightarrow$

$\left(3\cos\frac{4\pi}{3}\cdot\frac{12}{5}, 3\sin\frac{4\pi}{3}\cdot\frac{12}{5}\right) \Rightarrow \left(3\cos\frac{16\pi}{5}, 3\sin\frac{16\pi}{5}\right)$

$\left(3\cos\left(\frac{6\pi}{5}+2\pi\right), 3\sin\left(\frac{6\pi}{5}+2\pi\right)\right) \Rightarrow \left(3\cos\frac{6\pi}{5}, 3\sin\frac{6\pi}{5}\right)$

Normally we would reduce these to a fraction like $\frac{\sqrt{3}}{2}$ etc but since $\frac{6\pi}{5}$ is inconvenient we will leave it in this form. All other HW exercises come out conveniently.

(13) $\cos \frac{\pi t}{3} + 2k\pi = \cos \frac{5\pi t}{6}$ $\quad k \in J$

$\frac{\pi t}{3} + 2k\pi = \frac{5\pi t}{6}$

$2\pi t + 10k\pi = 5\pi t$ $\quad$ Let $k=1$ ie. one more

$12k\pi = 3\pi t$ $\quad$ revolution as per min t

$4k = t$ $\quad$ in #12

$(3\cos \frac{5\pi}{6} \cdot 4,\ 3\sin \frac{5\pi}{6} \cdot 4)$

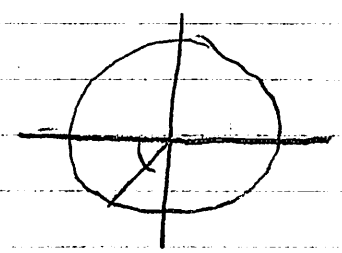
$(3\cos \frac{10\pi}{3},\ 3\sin \frac{10\pi}{3})$

$[(3\cos(2\pi + \frac{4\pi}{3})),\ 3\sin(2\pi + \frac{4\pi}{3})]$

$(3\cos \frac{4\pi}{3},\ 3\sin \frac{4\pi}{3})$

$(3 \cdot -\frac{1}{2},\ 3 \cdot -\frac{\sqrt{3}}{2})$

$(-\frac{3}{2},\ -\frac{3\sqrt{3}}{2})$



(14) $-\frac{\pi t}{12} = \frac{\pi 6}{4} + 2k\pi$ $\qquad$ $2k\pi - \frac{\pi 6}{12} = -\frac{\pi t}{4}$

$-\pi t = 3\pi t + 24k\pi$ $\qquad$ $24k\pi - \pi t = 3\pi t$

$-4\pi t = 24k\pi$ $\qquad$ $24k\pi = 4\pi t$

$t = -6k\pi$ $\quad k=1 \therefore t=6$ $\qquad$ $6k = t$

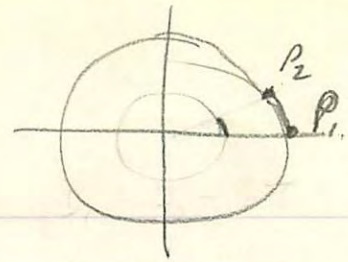
$3\cos -\frac{\pi}{12} \cdot 6$

$3\cos -\frac{\pi}{2} = 3\cos \frac{\pi}{2}$

$(0, -3)$ $\quad (3\cos -\frac{\pi}{2},\ 3\sin -\frac{\pi}{2})$

$(3 \cdot 0,\ 3 \cdot -1)$

$(0, -3)$.



(15) $P_1\ (3,0)$ $\qquad \omega_1 = \frac{\pi}{3}$

$P_2\ \left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ $\qquad \omega_2 = \frac{\pi}{4}$

$\frac{\pi}{6} \cdot 3 = \frac{\pi}{2}$

$$3\omega_1\tau + \frac{\pi}{2} = 3\omega_2\tau$$

$$\pi\tau + \frac{\pi}{2} = \frac{3\pi}{4}\tau$$

$$\tau - \frac{1}{2} = \frac{3}{4}\tau$$

$$\frac{\tau}{4} = \frac{1}{2}$$

$$\tau = 2$$

Location of coincidence

for P_1 use the following:

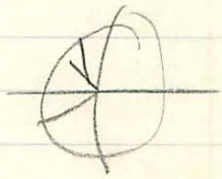
$a\cos(\omega\tau + b)$, $a\sin(\omega\tau + b)$

$\frac{4\pi\tau}{3} + \frac{3\pi}{2}$

$3\cos\left(\frac{\pi}{3}\cdot 2\right)$, $a\sin\left(\frac{\pi}{3}2\right)$

$3 \cdot \frac{-1}{2}$, $3 \cdot \frac{\sqrt{3}}{2}$

$\left(\frac{-3}{2}, \frac{3\sqrt{3}}{2}\right)$



For P_2 and ω_2. Since it has a head start you must use

$a\cos(\omega\tau + b)$, $a\sin(\omega\tau + b)$

NOTE $b = \frac{\pi}{6}$ since we do NOT have $a\cos(\omega\tau + Pb)$

$3\cos\left(\frac{\pi}{4}\cdot 2 + \frac{\pi}{6}\right)$, $3\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$

$3\cos\left(\frac{7\pi}{3}\right)$

Chapter 3
HONORS TRIGONOMETRY

T E A C U P S

Mr. Mumaugh

I A car wheel is rotating at 900 revolutions per minute.

A) Determine the angular velocity in radians per second on the tire tread, 35 cm from the center.
B) Determine the angular velocity in radians per second on the rim between the tire and the wheel, 20 cm from the center.
C) Find the following : The linear velocity in meters per second ...
   1) on the tire tread.
   2) on the rim between the tire and the wheel.
   3) at the center.
D) Determine the car's speed in km per hour. $\approx$ 119 kph

II A salt shaker is sitting 4 cm from the center of a lazy-susan. If the lazy-susan is turned through 120 º ...

A) Through how many radians does the shaker turn?
B) What distance does the shaker travel?
C) If the lazy-susan was turned that far in one-half second, what was the shakers angular velocity?
D) What was the shaker's linear velocity?

III Bull Dawg is riding on a ferris wheel. He notices that he is turning three revolutions per minute and that his seat is twenty-five feet from the axle of the wheel.

A) What is his angular velocity in revolutions per minute?
B) What is his angular velocity in degrees per minute?
C) What is his angular velocity in radians per minute?
D) What is his linear velocity in radians per minute?

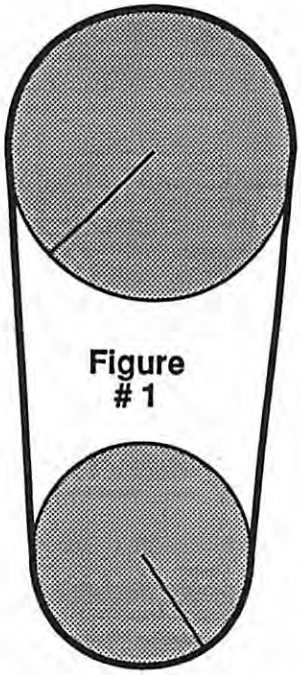


Figure # 1

IV A small pulley ~~6~~ 3 cm in diameter is connected by a belt to a larger pulley ~~15~~ $\frac{15}{2}$ cm in diameter, as shown in figure # 1.

A) The small pulley is driving at 120 rpm.
   1) Find the angular velocity of the drive pulley in radians per second.
   2) Find the linear velocity of the rim of the drive pulley.
   3) What is the linear velocity of the rim of the slave pulley? Explain.
   4) Find the angular velocity of the slave pulley in radians per second.
   5) How fast is the slave pulley turning in revolutions per minute?

B) The large pulley is driving at 120 rpm. ---- Answer the same 5 questions in part A.

V A large pulley has diameter 30 cm. The small pulley has a diameter of 8 cm.

A) Answer the same questions in IV A.

B) Answer the same questions in IV B.

VI The Courtship of Jake (the Snake) Jakester and the Czarina

In order to fully understand the particulars of Jake and the Czarina's love life, certain definitions need to be made:

- The Czarina's affection for (i.e. *interest in*) Jake is characterized by: $C(t) = \sin\left(\frac{\pi t}{10}\right) + \frac{1}{2}$
- Jake's affection for the Czarina is characterized by: $J(t) = -\cos\left(\frac{\pi t}{9}\right)$
- The time t is measured in days
- The Czarina and Jake met at $t = 0$. The Czarina liked Jake but he wasn't sure she was so nice -- at first.
- You "like" someone when you have a positive affection for him or her.
- You "dislike" someone when you have a negative affection for him or her.
- A couple is "happy" when the both like each other.
- A couple is "unhappy" when they both dislike each other.
- An individual is "happy" when he or she is liked by another.
- An individual is "unhappy" when he or she is disliked by another.

a) Who has the greater emotional range, Jake or the Czarina? Explain.

b) Whose emotions fluctuate more rapidly?

c) Graph Jake and the Czarina's first 31 days on the same axis.
(xmax ~~10π~~ 31, xmin 0, xstep 1 5, ymax 2 3, ymin -2,)

d) About which day were Jake and the Czarina most in love?

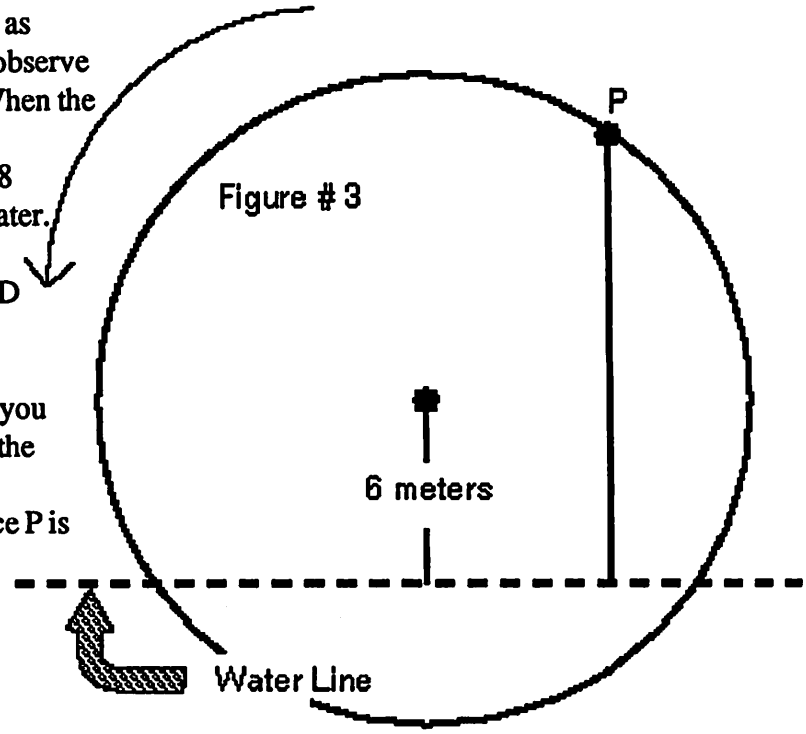
e) When does the Czarina like Jake intensely while Jake dislikes The Czarina intensely?

f) Describe how you can find, via the graphs, when Jake and the Czarina were happy as :

1) individuals

2) a couple

VII A water wheel, 14 meters in diameter is rotating as shown is figure # 3. You start a stop watch and observe the motion of point P on the rim of the wheel. When the stopwatch reads 3 seconds, P is at its maximum distance from the water. When the watch reads 8 seconds, P is at its maximum depth below the water.



A) Find the angular velocity of P in rpm AND in radians per second.

B) The distance of P from the water's surface varies with time. Plot the points you know. Sketch a cosine function through the points.

C) Write the equation expressing the distance P is from the water in terms of seconds t.

D) The general sinusoidal (look it up) equation is:

$$F(x) = A \cos(Bx - C) + D.$$

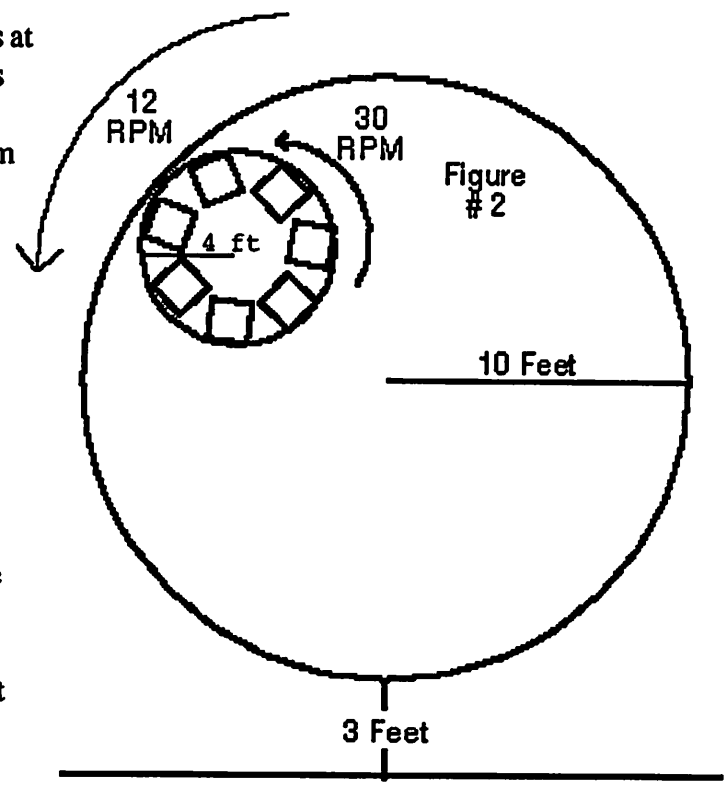
You know that A is amplitude,

D is axis movement, and the fraction $\frac{C}{B}$

is phase shift. Up till now, you have learned only that " B " " tells you" what the period is.

Now you know the real answer! Explain what " B " *really* is.

VIII One of the rides in Disneyland is called TEACUPS. It is a merry-go-round that rotates at a constant angular velocity while rings of seats rotate at a different (but constant) angular velocity. Suppose that the seats rotate at 30 rpm and the merry-go-round rotates at 12 rpm.



Peter Pan Ride (1 " E " ticket, please)

A) Find your linear velocity in feet per second due to the combined rotations of both wheels when your seat is:
   1. farthest from the center of the merry-go-round.
   2. nearest to the center of the merry-go-round.

B) In what direction are you actually moving when your seat is closest to the center of the merry-go-round?

C) As your ride continues, your distance from the Peter Pan ride varies. Suppose your seat is 23 feet from the fence at $t = 0$ seconds. Write an equation expressing your distance from the fence in terms of t.

D) Sketch your function.

Uniform Circular Motion.

TEACUPS DITTO

①

ⓐ Since the angular velocity is 900 RPM and since there are 2π radians in one revolution and 60 seconds in one minute,

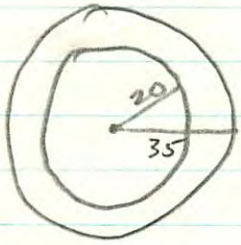
$$T = \frac{1}{900} \therefore \omega = \frac{2\pi}{\frac{1}{900}} \cdot \frac{1}{60} = 30\pi \text{ RADIANS per second on the Tread.}$$

Consider

$$\omega = 900 \frac{\cancel{Rev}}{\cancel{min}} \cdot 2\pi \frac{RAD}{\cancel{Rev}} \cdot \frac{1}{60} \frac{\cancel{min}}{sec}$$

b) Since the angular velocity of ANY point on the same rotating object or Ray has the same angular velocity, ie $\omega = 30\pi$ Rad/sec at the Rim.

ⓒ



linear velocity $= r\omega \therefore$
$35 \cdot 30\pi = 1050\pi$ cm per second.
centimeters per second.

C1 So $rv = 1050\pi \frac{cm}{sec} \cdot \frac{1 \text{ meter}}{100 cm} = 10.5\pi$ meters/sec at the tread

C2 $r = 20 \therefore$
$20 \cdot 30\pi \cdot \frac{1}{100} = 6\pi$ m/sec at the Rim

C3 $r = 0 \therefore 0 \cdot 30\pi \frac{1}{100} = 0$ m/sec at the center.

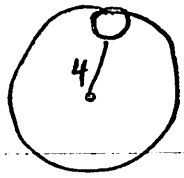
It may appear that the center is not moving. ie the center of the wheel is stopped and still it is rotating at 900 RPM.

D. We know that the linear velocity is 10.5π m/sec so all that is needed is to convert from meters per second to kilometers per hour. So:

Dimensional analysis

$$10.5\pi \frac{m}{sec} \cdot \frac{1 \text{ KM}}{1000 \text{ m}} \cdot \frac{3600 \text{ sec}}{\text{hour}} \approx 33 \cdot \frac{3600}{1000} = 118.8$$

Uniform Circular motion (Page 2)
(Tea cups)

(2) 

a) $120° \Rightarrow 1° = \frac{\pi}{180}$ Rad $\therefore 120° = \frac{2\pi}{3}$ Rad

b) arc length is Radius · Radian (on unit circle) $\therefore$

$4 \cdot \frac{2\pi}{3} = \frac{8\pi}{3}$

c) $120° \equiv \frac{1}{2}$ Sec. 1 Rev $\equiv 360° \therefore T = \frac{3}{2}$ sec.

$\omega = \frac{2\pi}{T} \therefore \frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$

d) Linear Velocity $= r\omega \therefore 4 \cdot \frac{4\pi}{3} = \frac{16\pi}{3}$ cm/sec.

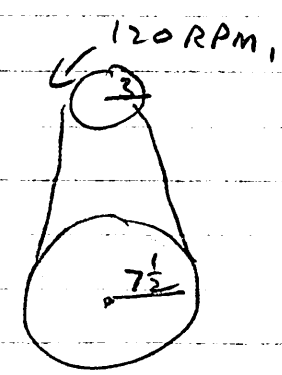
(3) a) $\frac{1 \text{ Rev}}{20 \text{ Sec}} \cdot \frac{60 \text{ Sec}}{\text{min}} = 3$ RPM

b) $\frac{3 \text{ Rev}}{\text{min}} \cdot \frac{360 \text{ Deg}}{\text{Rev}} = 1080$ Deg/min.

c) $\frac{3 \text{ Rev}}{\text{min}} \cdot \frac{2\pi \text{ Rad}}{1 \text{ Rev}} = 6\pi$ Rad/min

D) $6\pi \cdot 25 = 150\pi$ ft/min.

(4) a) $\frac{120 \text{ Rev}}{\text{min}} \cdot \frac{2\pi \text{ Rad}}{\text{Rev}} \cdot \frac{1 \text{ min}}{60 \text{ Sec.}} = 4\pi$ Rad/sec.



b) $3 \text{ cm} \cdot 4\pi/\text{sec} = 12\pi \frac{\text{cm}}{\text{sec}}$.

c) 12π cm/sec Pulley's are connected but the belt $\therefore$ Belt is traveling at the same rate all the time it around small wheel or around large Pulley.

D) $v = r\omega \Rightarrow \omega = \frac{v}{r} \Rightarrow \frac{\overset{4}{\cancel{12}}\pi \text{ cm}}{\text{sec}} \cdot \frac{2}{\underset{5}{\cancel{15}}\text{ cm}} = \frac{8\pi}{5}$ Rad/sec

E) $\frac{\overset{4}{\cancel{8\pi}} \text{ Rad}}{\cancel{5 \text{ Sec}}} \cdot \frac{1 \text{ Rev}}{\cancel{2\pi \text{ Rad}}} \cdot \frac{\overset{12}{\cancel{60 \text{ Sec}}}}{\text{min}} = 48$ RPM

(Tea cups)

(5) a) $\frac{45 \text{ Rev}}{\text{min}} \cdot \frac{2\pi \text{ Rad}}{1 \text{ Rev}} = 90\pi \frac{\text{RAd}}{\text{min}}$

450
90

b) $r\omega \Rightarrow 15 \cdot 90\pi \frac{\text{RAd}}{\text{min}} = 1350\pi \frac{\text{cm}}{\text{min}}$.

c) linear velocity of small gear is also 1350π cm/min

D) $1350\pi \frac{\text{cm}}{\text{min}} = \underset{\text{cm.}}{4} \cdot X$, $X = \frac{675\pi}{2}$ RAd/min.

E) $\frac{675\pi \text{ Rad}}{2 \text{ min}} \cdot \frac{1 \text{ Rev}}{2\pi \text{ RAd}} = \frac{675}{4} \frac{\text{Rev}}{\text{min}}$.

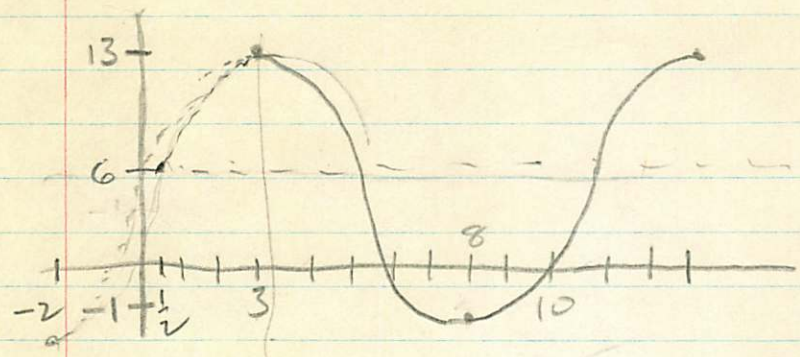
$$f(x) = a\cos B\left(t + \frac{c}{\omega}\right) + d$$

(Teacups)

(7) $T = 10$ sec. ie time for one Rev.

$\frac{2\pi}{10} = \frac{\pi}{5} = \omega$

A) $\frac{\pi \text{ rad}}{5 \text{ sec.}} \cdot \frac{\text{Rev}}{2\pi \text{ rad}} = \frac{1}{10}$ Rev sec.



$\cancel{y = 6 \cos\left(\frac{\pi}{5}(t-3)\right)}$ $\quad 7 \sin \frac{\pi}{5}\left(x - \frac{1}{2}\right) + 6$

B is the Angular velocity.

written in cosine.

$\frac{2\pi}{x} = 10$

$\therefore x = \omega$

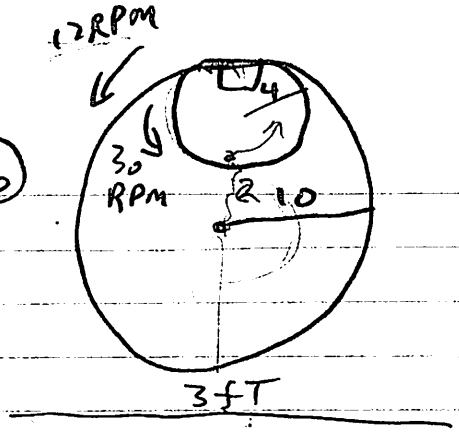
$7 \cos \frac{\pi}{5}(x-3) + 6$

OR

$7 \sin \frac{\pi}{5}\left(x - \frac{1}{2}\right) + 6 = 7 \sin\left(\frac{\pi}{5}x - \frac{\pi}{10}\right) + 6$

(Teacups)

(6)



Large wheel: $\frac{12 \text{ Rev}}{} \equiv 1 \text{ min}$ ∴

$T = 1 \text{ Rev} \equiv \frac{1}{12} \text{ min} = 5 \text{ sec}$

$\frac{2\pi}{T} = \frac{2\pi}{5 \text{ sec}} = \omega$

Linear velocity of ~~farthest~~ from center ⇒ $r = 10$ ∴ linear velocity is $\frac{2\pi}{5} \cdot 10 = 4\pi$.

Linear velocity nearest to center is $\frac{2\pi}{5} \cdot 2 = \frac{4\pi}{5}$

Small wheel 30 RPM ⇒ 30 Rev ≡ 1 min

$1 \text{ Rev} = \frac{1}{30} \text{ min} = 2 \text{ sec}$.

$\frac{2\pi}{2 \text{ sec}} = \pi/\text{sec}$ Linear velocity is 4π (& ω)

A) ① Combined effect farthest from center is sum of Both ∴ $4\pi + 4\pi = 8\pi/\text{sec}$.

b → ② Combined effect nearest to center is difference. Notice effect is "backward" movement ∴

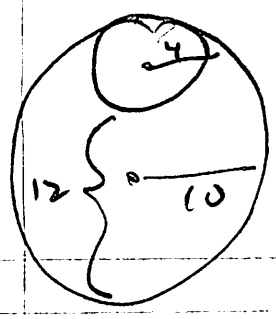
$\frac{4\pi}{5} - 4\pi = -\frac{16\pi}{5}$.

c)

Effect of Lg wheel — Effect of Small wheel — mid pt Between max & min

① $y = \cos(\_\_\_ + \cos \_\_\_ + 13$

↓ cont Next pg.



The effect of small wheel accounts for 4 feet of Distance.
The effect of Lg wheel accounts for 6 feet of Distance.

Since value of $|\cos| \leq 1$ in order to create the effect trigonometrically we need to change the amplitude to:

$$Y = \underline{6 \cos \_\_} + \underline{4 \cos \_\_} + 13$$

This is a fctn of time ∴

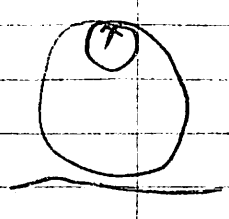
$$Y = \overbrace{6 \cos \_\_ \tau}^{\text{Lg wheel}} + \overbrace{4 \cos \_\_ \tau}^{\text{small}} + \overset{\text{axis}}{13}$$

usually what we have here is the fctn's rotational velocity. $\frac{2\pi}{\omega} = 2\pi$

from pg 4 we see the period, 1 rev
Big wheel 1 Rev ≡ 5 sec. ∴ 1 Rev = 2π

Large wheel 30 Rev = 1 min
1 Rev = 2 sec. = π
$\omega_L = \pi$

Small wheel 12 Rev = 60
$\omega_s = \frac{2\pi}{5}$ 1 rev = $\frac{1}{5}$ sec

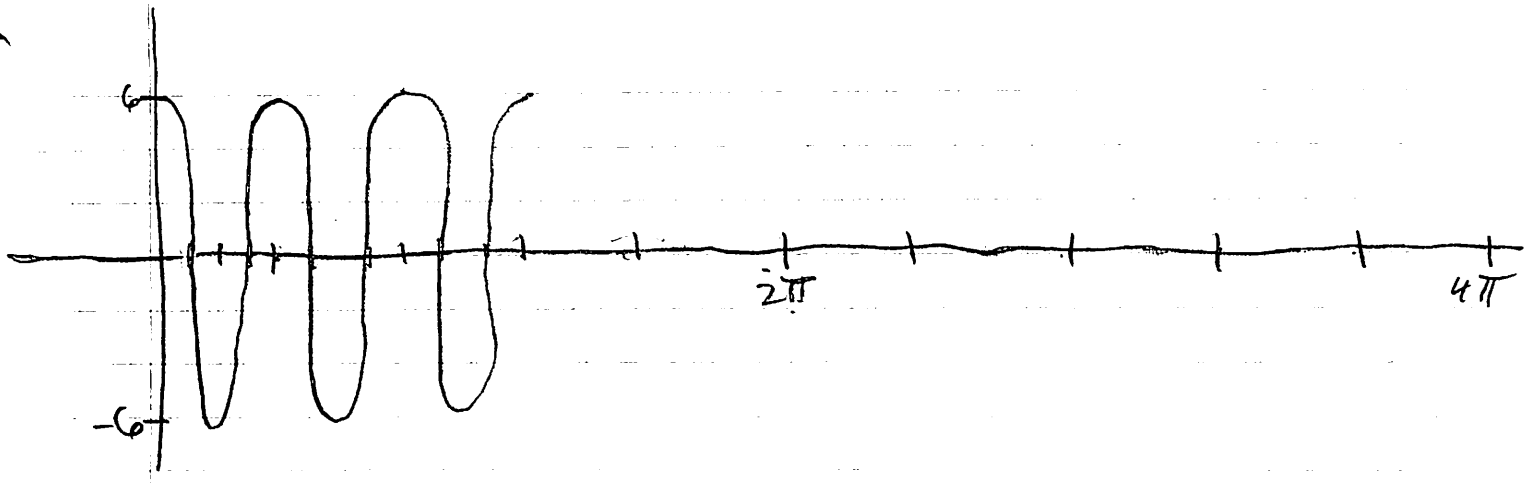


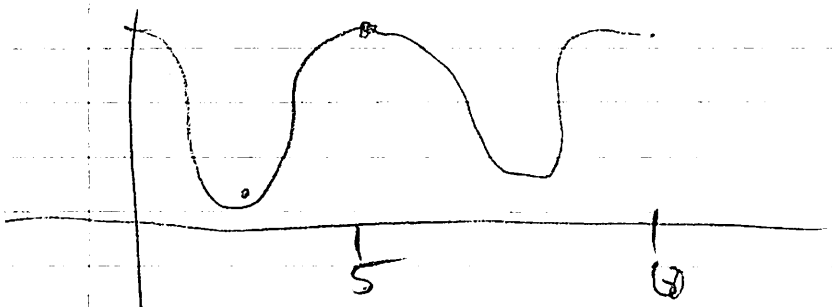
Both start at top of circle ∴ No phase shift.

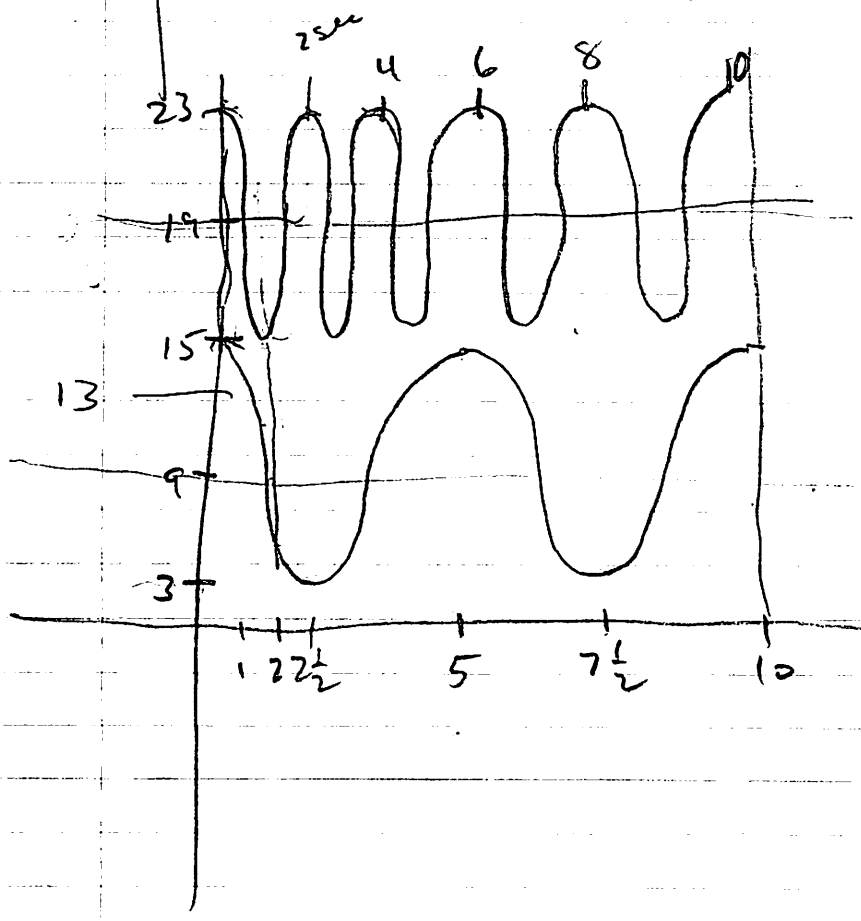
why not? Seems to me take $\frac{\pi}{2}$ shift for both wheels.

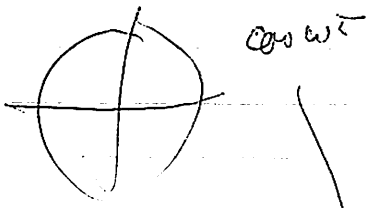
$$Y = 6 \cos \pi\tau + 4 \cos \frac{\pi}{5}\tau + 13$$

(Teacups)









5
Fctn
General Period length
$\frac{2\pi}{5}$

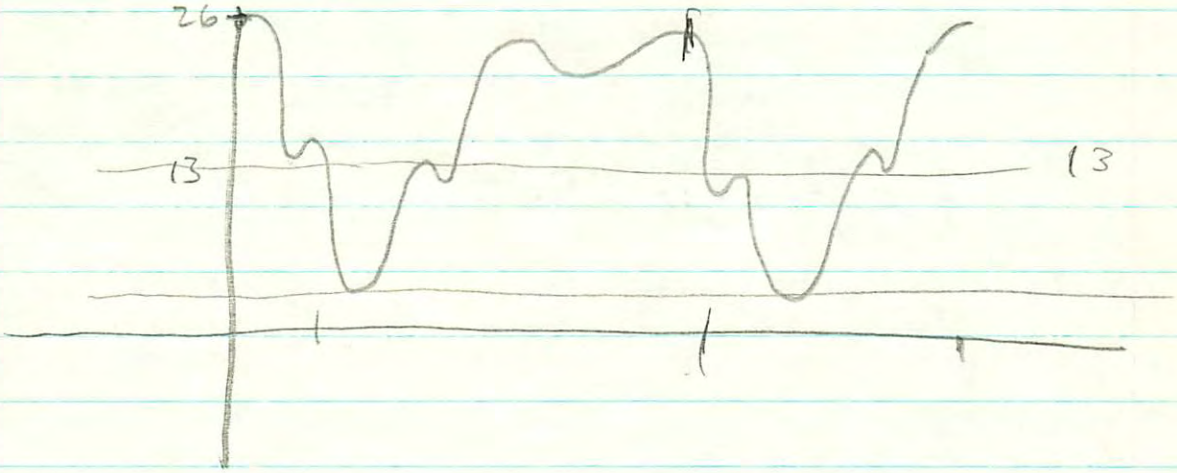
$\frac{2\pi}{2}$ ~~Length~~ Time for 1 Rev

#6

actual "rotations" of "cup" is
The cup ω plus ω for Saucer.
thus the axis of The cup is
with respect to The Saucer Not Peter Pan.

$$f(t) = 6\cos\frac{2\pi}{5}t + 4\cos\left(\frac{2\pi}{5}+\pi\right)t + 13$$

$$6\cos\frac{2\pi}{5}t + 4\cos\left(\frac{2\pi}{5}t\right) + 13$$



PT on big circle to fence

$10\sin(24\pi t + \frac{\pi}{2}) + 10 + 3$

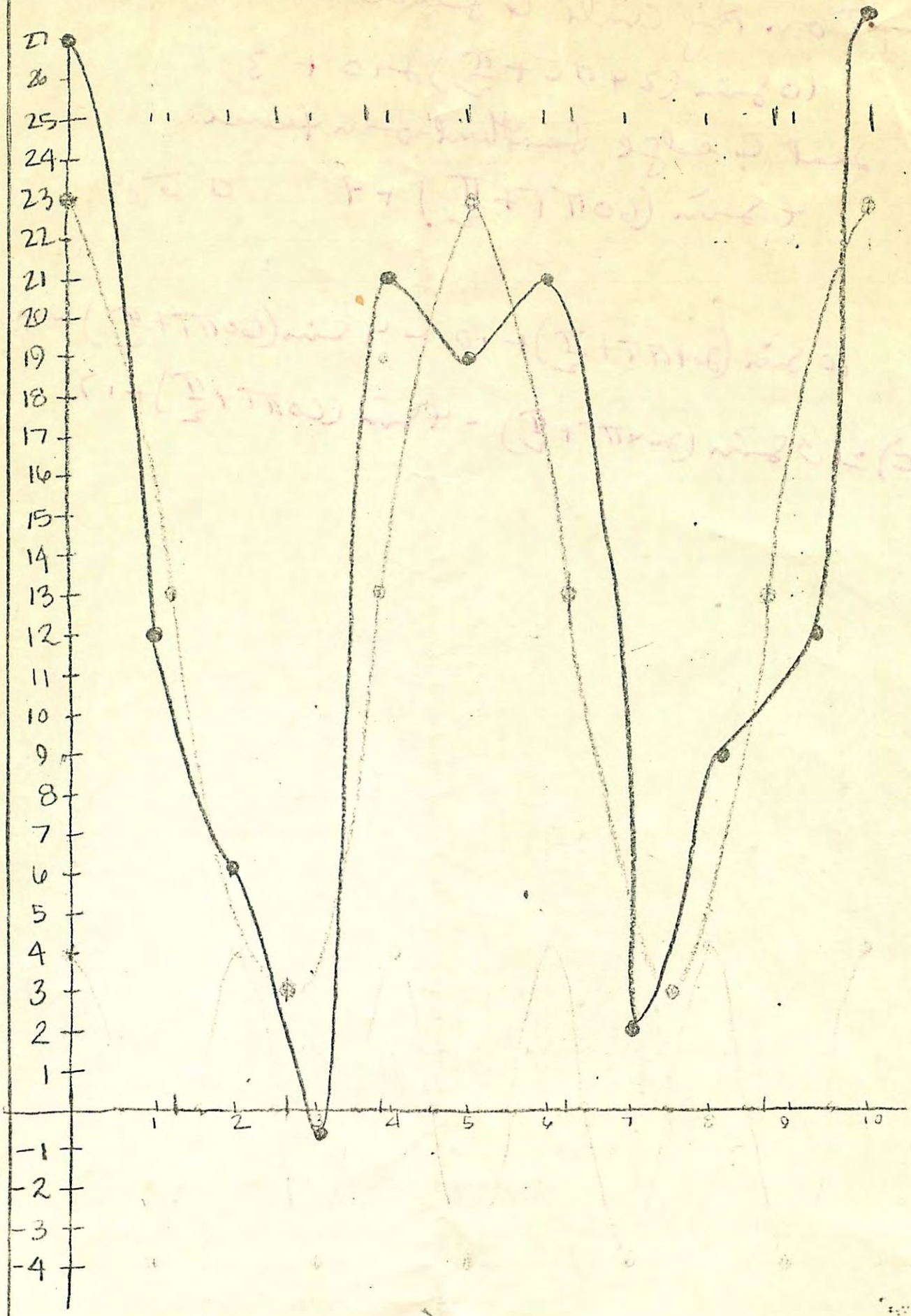
seat to edge farthest from fence

$4\sin(60\pi t + \frac{\pi}{2}) + 4$ 0 to 8

$10\sin(24\pi t + \frac{\pi}{2}) + 10 - 4\sin(60\pi t + \frac{\pi}{2}) - 4 + 3$

$f(t) = 10\sin(24\pi t + \frac{\pi}{2}) - 4\sin(60\pi t + \frac{\pi}{2}) + 17$

K Green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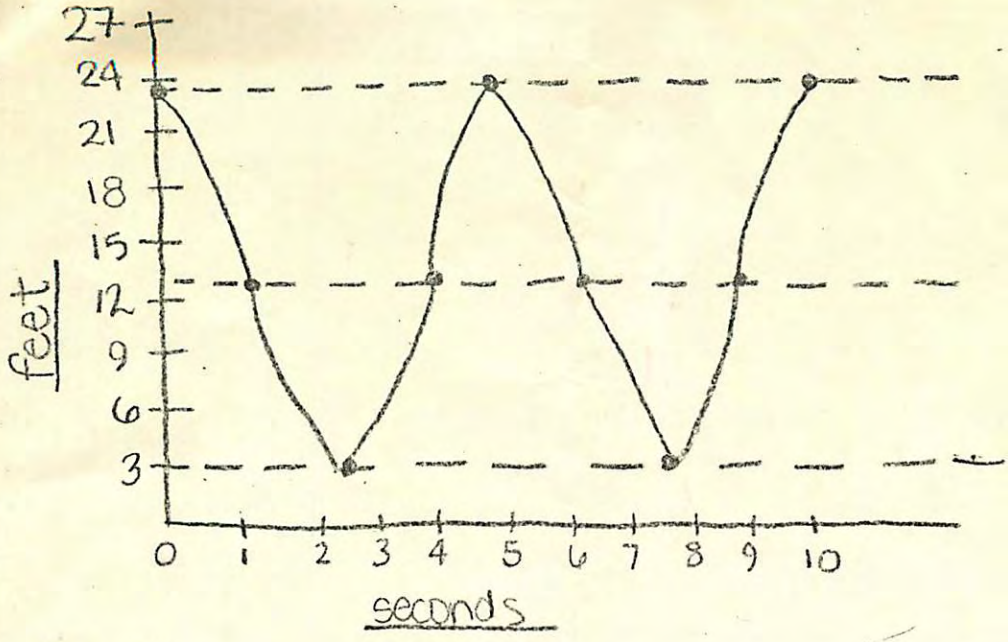
Karin Green

c (continued):

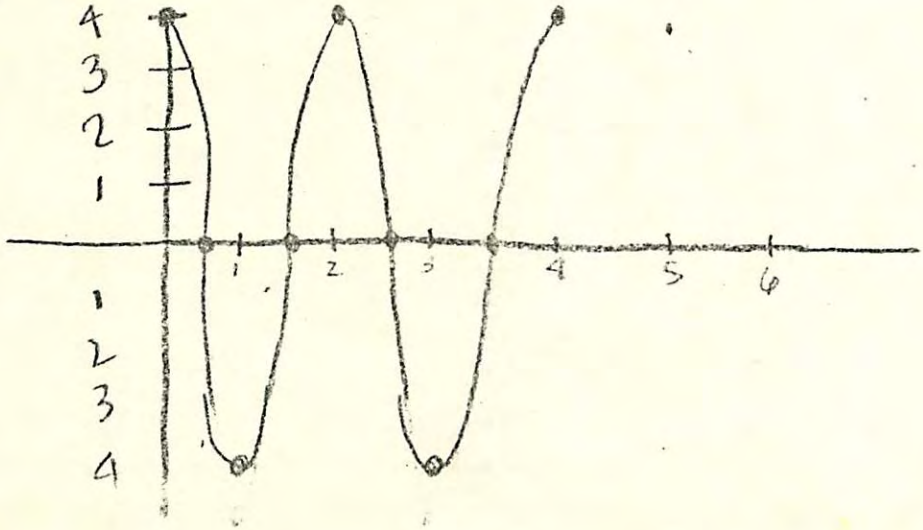


merry go round

$a = 10$
$d = +13$
$ph = 0$

$\boxed{\text{function: } y = 10 \cos \frac{2\pi}{5} t + 13}$

$y = a \cos wt + d$



$a = 4$
$d = 0$
$ph = 0$

$y = 4 \cos \pi t$

Karin Green
10·23·84

6) 30 rev = 1 min
30 rev = 60 sec
1 rev = 2 sec
T = 2 sec

12 rev = 1 min
12 rev = 60 sec
1 rev = 5 sec
T = 5 sec

$$\left.\begin{aligned} \omega_S &= \frac{2\pi}{2} = \pi \\ \omega_L &= \frac{2\pi}{5} \end{aligned}\right\} \text{angular velocity}$$

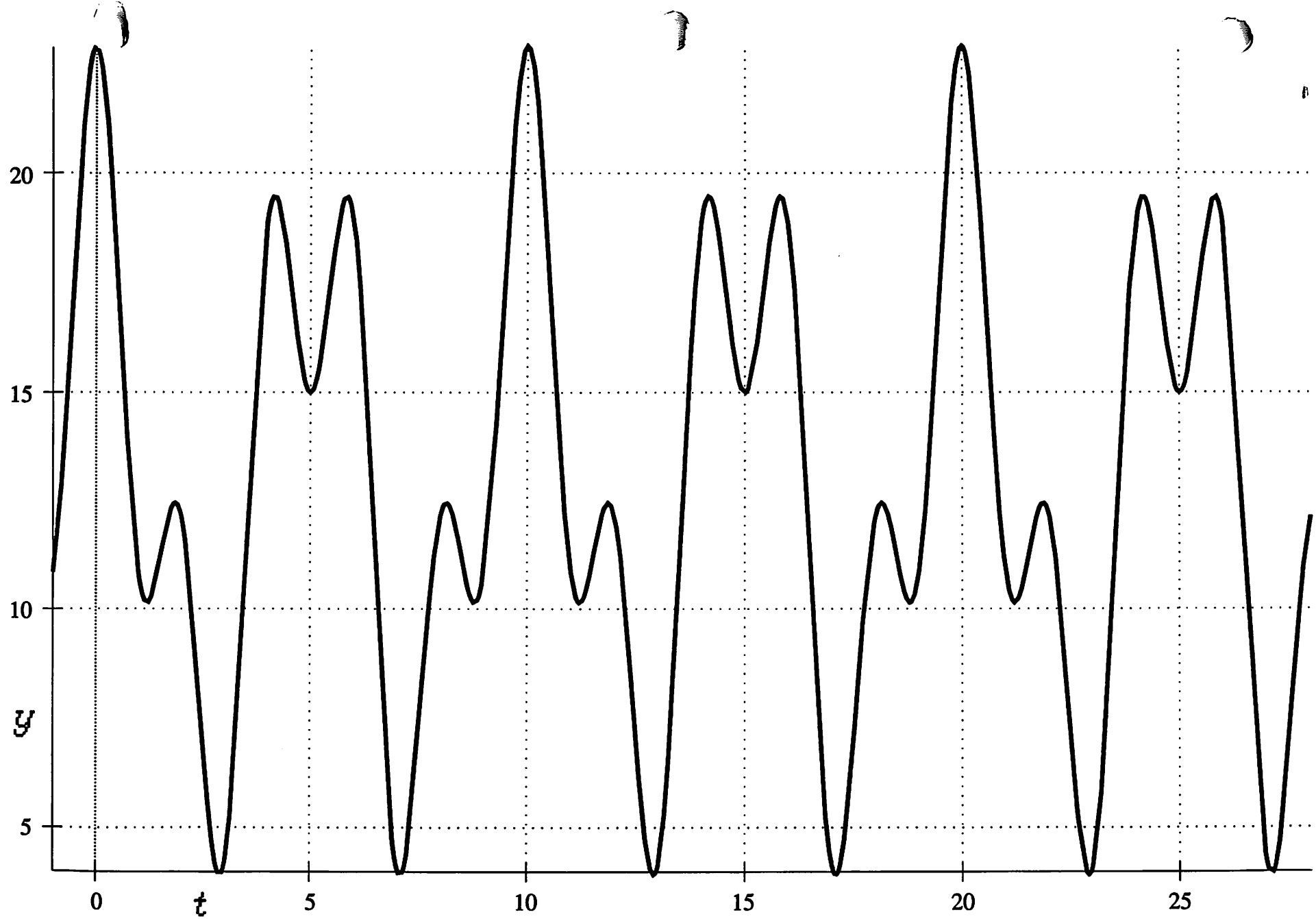
$$\left.\begin{aligned} pw_S &= 4 \cdot \pi = 4\pi \text{ ft per sec} \\ pw_{NL} &= 2 \cdot \frac{2\pi}{5} = \frac{4\pi}{5} \text{ ft per sec} \\ pw_{FL} &= 10 \cdot \frac{2\pi}{5} = 4\pi \text{ ft per sec} \end{aligned}\right\} \text{linear velocity}$$

A) ① $pw_S + pw_{FL} = 4\pi + 4\pi = 8\pi$ ft per sec
② $pw_S + pw_{NL} = 4\pi + \frac{4\pi}{5} = \frac{24\pi}{5}$ ft per sec

B) clockwise

C) <u>SINUSOIDALLY</u> ~ of, relating to, shaped like, or varying according to a sine curve or sine wave

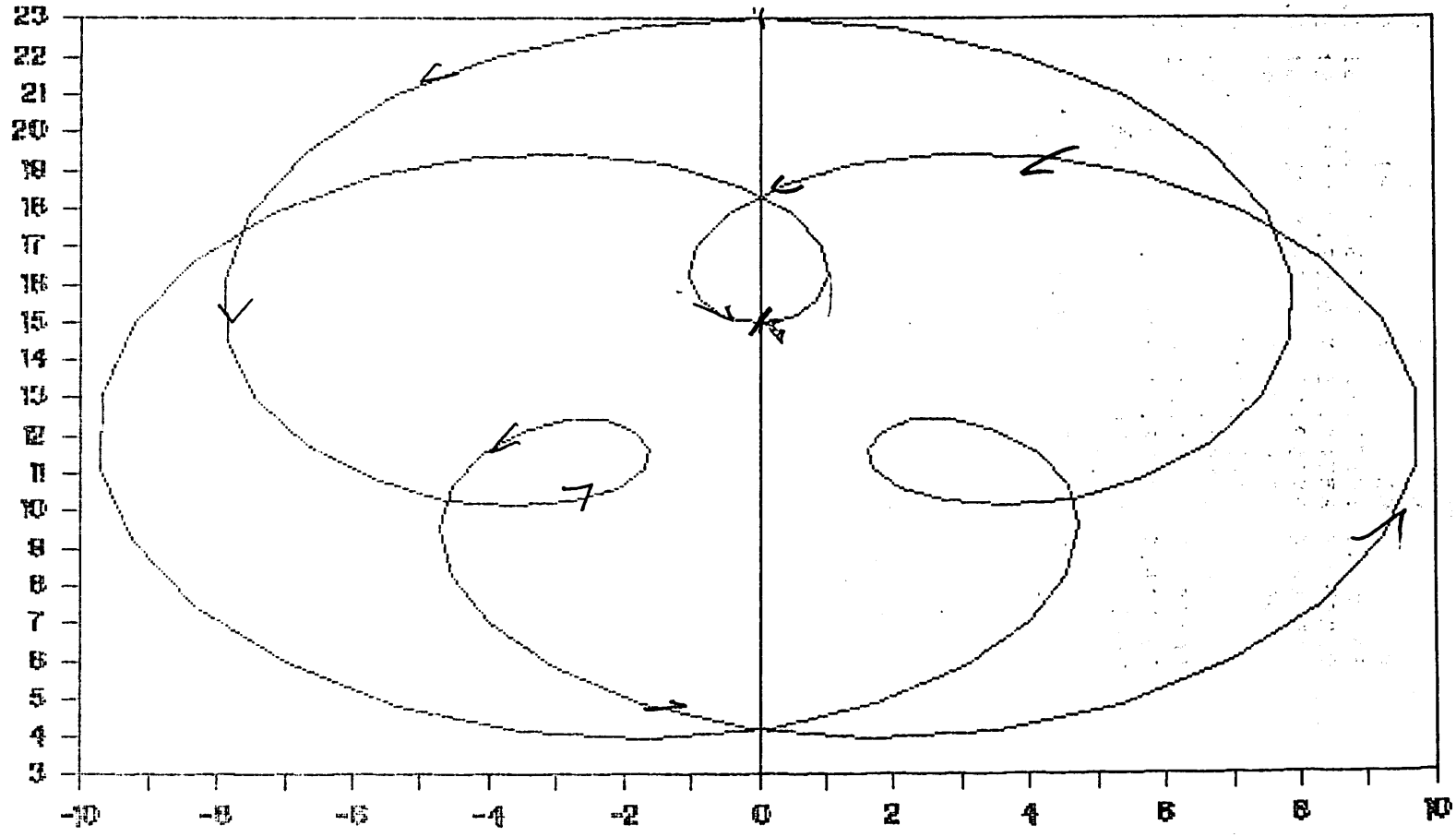
+10



$$Teacup(t) = 6\cos\frac{2\pi}{5}t + 4\cos\pi t + 13$$

T	A	B	d	x
0	19	4	23	0
0.1	18.95268	3.804226	22.75691	1.988067
0.2	18.81149	3.236067	22.04756	3.843280
0.3	18.57865	2.351141	20.92979	5.444815
0.4	18.25784	1.236067	19.49390	6.694748
0.5	17.85410	-1.4E-15	17.85410	7.526711
0.6	17.37381	-1.23606	16.13774	7.911508
0.7	16.82454	-2.35114	14.47340	7.859147
0.8	16.21496	-3.23606	12.97889	7.417108
0.9	15.55467	-3.80422	11.75044	6.665030
1	14.85410	-4	10.85410	5.706339
1.1	14.12428	-3.80422	10.32006	4.657655
1.2	13.37674	-3.23606	10.14067	3.637019
1.3	12.62325	-2.35114	10.27211	2.752092
1.4	11.87571	-1.23606	10.63964	2.089497
1.5	11.14589	5.6E-15	11.14589	1.706339
1.6	10.44532	1.236067	11.68139	1.624736
1.7	9.785039	2.351141	12.13618	1.829899
1.8	9.175456	3.236067	12.41152	2.271938
1.9	8.626188	3.804226	12.43041	2.871214
2	8.145898	4	12.14589	3.526711
2.1	7.742159	3.804226	11.54638	4.126590
2.2	7.421341	3.236067	10.65740	4.559888
2.3	7.188501	2.351141	9.539642	4.728207
2.4	7.047311	1.236067	8.283379	4.556225
2.5	7	-1.7E-14	7	4
2.6	7.047311	-1.23606	5.811243	3.052226
2.7	7.188501	-2.35114	4.837360	1.743928
2.8	7.421341	-3.23606	4.185273	0.142393
2.9	7.742159	-3.80422	3.937933	-1.65445
3	8.145898	-4	4.145898	-3.52671
3.1	8.626188	-3.80422	4.821962	-5.34335
3.2	9.175456	-3.23606	5.939388	-6.97422
3.3	9.785039	-2.35114	7.433898	-8.30203
3.4	10.44532	-1.23606	9.209256	-9.23318
3.5	11.14589	2.8E-14	11.14589	-9.70633
3.6	11.87571	1.236067	13.11178	-9.69794
3.7	12.62325	2.351141	14.97439	-9.22422
3.8	13.37674	3.236067	16.61281	-8.33930
3.9	14.12428	3.804226	17.92851	-7.12979
4	14.85410	4	18.85410	-5.70633
4.1	15.55467	3.804226	19.35890	-4.19289
4.2	16.21496	3.236067	19.45102	-2.71482
4.3	16.82454	2.351141	19.17568	-1.38701
4.4	17.37381	1.236067	18.60987	-0.30305
4.5	17.85410	-1.1E-14	17.85410	0.473288
4.6	18.25784	-1.23606	17.02177	0.913704
4.7	18.57865	-2.35114	16.22751	1.027320
4.8	18.81149	-3.23606	15.57543	0.859001
4.9	18.95268	-3.80422	15.14846	0.484068
5	19	-4	15	6.6E-15
5.1	18.95268	-3.80422	15.14846	-0.48406
5.2	18.81149	-3.23606	15.57543	-0.85900
5.3	18.57865	-2.35114	16.22751	-1.02732
5.4	18.25784	-1.23606	17.02177	-0.91370

5.5	17.85410	-3.3E-14	17.85410	-0.47328
5.6	17.37381	1.236067	18.60987	0.303056
5.7	16.82454	2.351141	19.17568	1.387011
5.8	16.21496	3.236067	19.45102	2.714826
5.9	15.55467	3.804226	19.35890	4.192894
6	14.85410	4	18.85410	5.706339
6.1	14.12428	3.804226	17.92851	7.129791
6.2	13.37674	3.236067	16.61281	8.339301
6.3	12.62325	2.351141	14.97439	9.224228
6.4	11.87571	1.236067	13.11178	9.697949
6.5	11.14589	7.8E-14	11.14589	9.706339
6.6	10.44532	-1.23606	9.209256	9.233188
6.7	9.785039	-2.35114	7.433898	8.302035
6.8	9.175456	-3.23606	5.939388	6.974220
6.9	8.626188	-3.80422	4.821962	5.343350
7	8.145898	-4	4.145898	3.526711
7.1	7.742159	-3.80422	3.937933	1.654454
7.2	7.421341	-3.23606	4.185273	-0.14239
7.3	7.188501	-2.35114	4.837360	-1.74392
7.4	7.047311	-1.23606	5.811243	-3.05222
7.5	7	-1.2E-13	7	-4
7.6	7.047311	1.236067	8.283379	-4.55622
7.7	7.188501	2.351141	9.539642	-4.72820
7.8	7.421341	3.236067	10.65740	-4.55988
7.9	7.742159	3.804226	11.54638	-4.12659
8	8.145898	4	12.14589	-3.52671
8.1	8.626188	3.804226	12.43041	-2.87121
8.2	9.175456	3.236067	12.41152	-2.27193
8.3	9.785039	2.351141	12.13618	-1.82989
8.4	10.44532	1.236067	11.68139	-1.62473
8.5	11.14589	1.6E-13	11.14589	-1.70633
8.6	11.87571	-1.23606	10.63964	-2.08949
8.7	12.62325	-2.35114	10.27211	-2.75209
8.8	13.37674	-3.23606	10.14067	-3.63701
8.9	14.12428	-3.80422	10.32006	-4.65765
9	14.85410	-4	10.85410	-5.70633
9.1	15.55467	-3.80422	11.75044	-6.66503
9.2	16.21496	-3.23606	12.97889	-7.41710
9.3	16.82454	-2.35114	14.47340	-7.85914
9.4	17.37381	-1.23606	16.13774	-7.91150
9.5	17.85410	-2.0E-13	17.85410	-7.52671
9.6	18.25784	1.236067	19.49390	-6.69474
9.7	18.57865	2.351141	20.92979	-5.44481
9.8	18.81149	3.236067	22.04756	-3.84328
9.9	18.95268	3.804226	22.75691	-1.98806
10	19	4	23	-3.5E-13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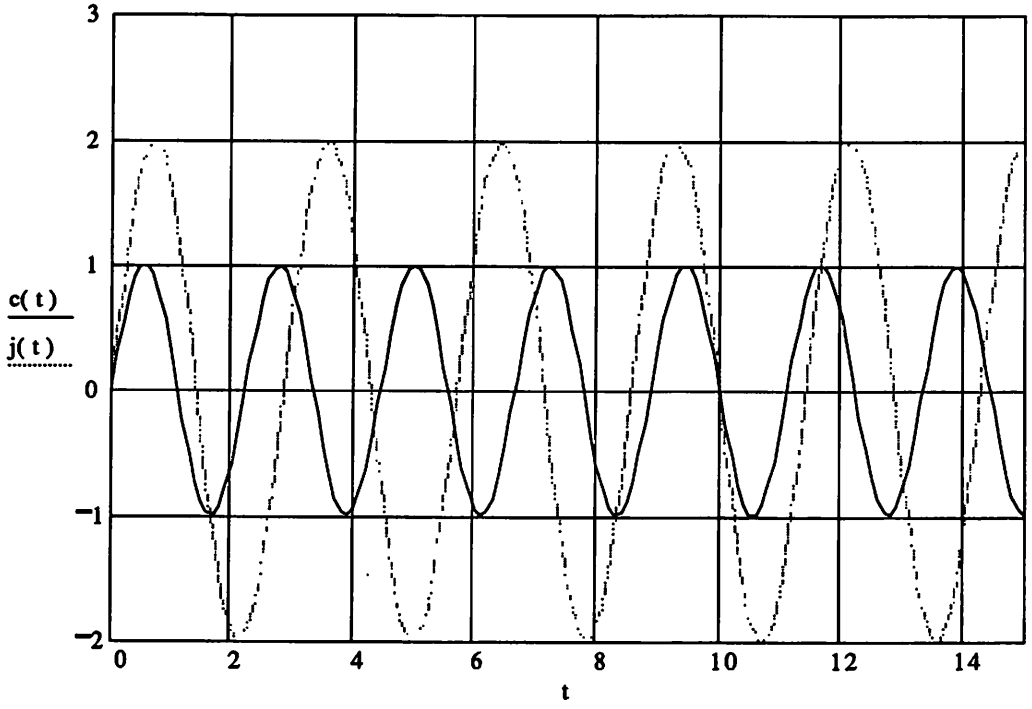
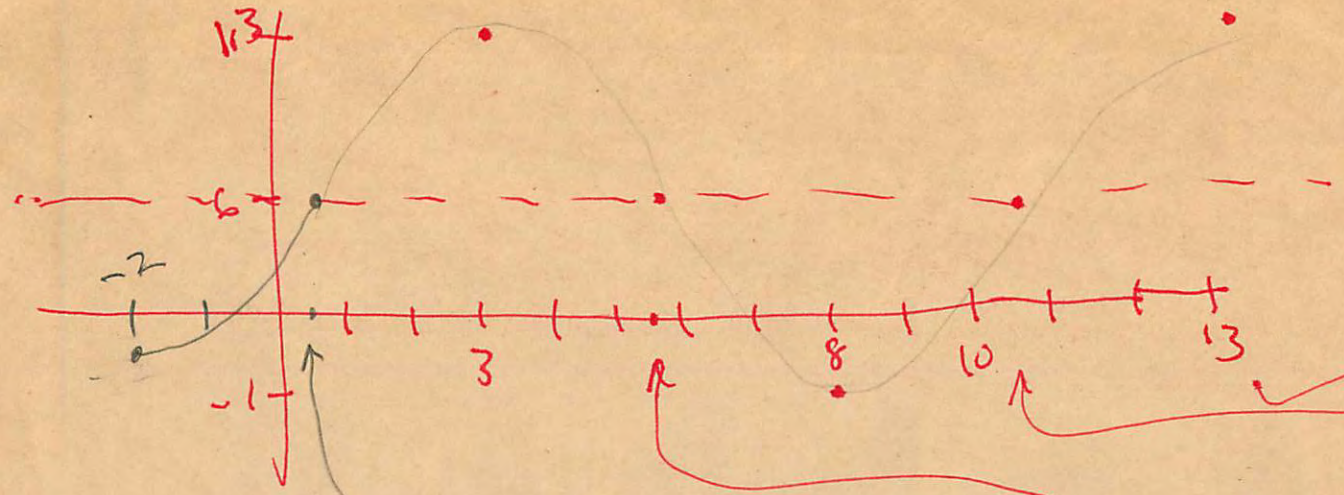
Jake and the Czarina

$t := 0.0, 0.1 .. 15$

$c(t) := \sin\left(\frac{9 \cdot \pi \cdot t}{10}\right) \qquad j(t) := 2 \cdot \sin\left(\frac{7 \cdot \pi \cdot t}{10}\right)$





and $10\frac{1}{2}$ and 13

$\frac{1}{2}$ way between $3-8$ $\frac{5}{2}+3=5\frac{1}{2}$

where is next fctn axis is at $\tau=\frac{1}{2}$ and the one before

that is at $\tau=-2$

#1 must solve for b 1st

$E = E_m \sin(\omega t + b) \quad -\frac{\pi}{2} \le b \le \frac{\pi}{2}$

but $E = E_m = 150$ when $t = \frac{1}{2}$ $\quad \omega = \frac{2\pi}{\frac{1}{60}} = 120\pi$

$150 = 150 \sin\left(\frac{120\pi}{2} + b\right)$

$1 = \sin(60\pi + b)$

$1 = \sin b \Rightarrow b = \frac{\pi}{2}$

$t = \frac{1}{360} \qquad E = 150 \sin\left(\frac{120\pi}{360} + \frac{\pi}{2}\right)$

$= 150 \sin\left(\frac{\pi}{3} + \frac{\pi}{2}\right)$

$= 150 \sin\left(\frac{5\pi}{6}\right)$

$= 150 \cdot \frac{1}{2} = 75$

#2 $\quad E = 150 \sin\left(\frac{120}{60}\pi + \frac{\pi}{2}\right)$

$= 150 \sin\left(\frac{3\pi}{2} + \frac{\pi}{2}\right) = 150 \sin 2\pi = 0.$

#3 $E = E_m \sin \omega t. \qquad \frac{2\pi}{\frac{1}{50}} = 100\pi = \omega$

$E_m = 140$

$E = 140 \sin \frac{100\pi}{120}$

$= 140 \sin \frac{5\pi}{6}$

$= 140 \cdot \frac{1}{2} = 70$

#4

$E = 140 \sin \frac{100\pi}{60}$

$E = 140 \sin \frac{5\pi}{3}$

$E = 140 \cdot \frac{-\sqrt{3}}{2} = -70\sqrt{3}$

(5) Particle in Simple harmonic motion is given by
$y = a\cos(\omega t + b)$ or $y = a\sin(\omega t + b)$
Period $2\pi \Rightarrow \omega = 1$
$y = a\cos(t+b)$ is max when $t+b = 0$. Since $t=0$ when it is max. b also $=0$.
Max value is 4 $\therefore a = 4$.

$y = 4\cos t$

(6) $y = a\cos(t+b)$
$\sqrt{3} = 2\cos(t+b)$
$\frac{\sqrt{3}}{2} = \cos(t+b) \Rightarrow t+b = \frac{\pi}{6}$ but $t=0 \therefore b = \frac{\pi}{6}$

$y = 2\cos\left(t + \frac{\pi}{6}\right)$

(7) $0 = a\sin(\omega t + b) \Rightarrow \omega t + b = 0$

(1) $v = a\omega\cos 0 \Rightarrow v = a\omega$.

$a = a\sin(\omega t + b) \Rightarrow \omega t + b = \frac{\pi}{2}$
(2) $v = a\omega\cos\frac{\pi}{2} = 0$

(3) max velocity occurs when $\omega t + b = 0 \therefore t = \frac{-b}{\omega}$

(4) min velocity occurs when $\omega t + b = \pi$ $t = \frac{\pi - b}{\omega}$

Extending 3 and 4 to general case $t = \frac{-b + 2k\pi}{\omega}$
and $t = \frac{(2k+1)\pi - b}{\omega}$ respectively.

128

(8) $y = \sqrt{3} \sin 4t$

$v = \sqrt{3} \cos 4t$

$v = \sqrt{3} \cos 4 \cdot \frac{\pi}{12}$

$= \sqrt{3} \cos \frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

max velocity $\sqrt{3}$ when $\cos 4t = 1$.

15 min

Honors Trigonometry

El Quizzo # 1

Name _____

Per _____ Date 10/23/97

For each of the following, state Range, Domain, Period, Phase shift (including direction) and amplitude. Graph on the supplied axis.

1) $F(x) = \sec\left(\frac{2}{3}x + \frac{\pi}{6}\right) + 1$

$\sec \frac{2}{3}\left(x + \frac{\pi}{4}\right) + 1$

$\frac{2}{3}\left(x + \frac{\pi}{4}\right) = \frac{(2k+1)\pi}{2}$ $\frac{3}{2}$

$x + \frac{\pi}{4} = \frac{6k\pi}{4} + \frac{3\pi}{4}$

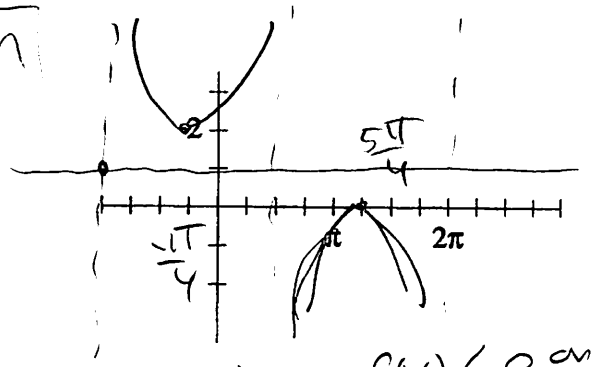
$= \frac{3k\pi}{2} + \frac{\pi}{2}$

$x = \frac{(3k+1)\pi}{2}$

$k = -1$ $-\pi$

$= 0$ $\frac{\pi}{2}$

1 2π



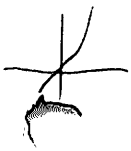
D'_f $f(x) \le 0$ or $f(x) \ge 1$

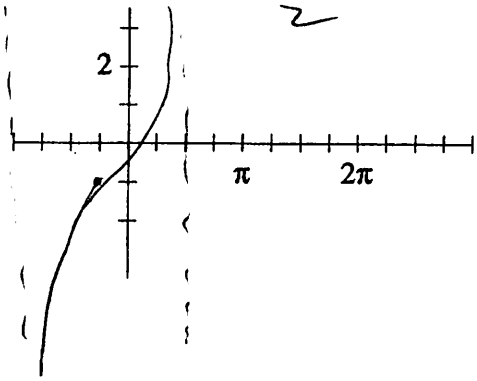
R_f $x \ne \left\{\frac{(3k+1)\pi}{2}\right\}$

Per $\frac{2\pi}{\frac{2}{3}} = 3\pi$

PS $\frac{\pi}{4}$ Left

Amp 1





2) $F(x) = \tan\left(\frac{2}{3}x + \frac{\pi}{6}\right) - 1$

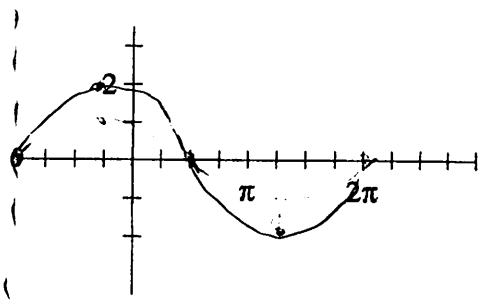
R_f $f(x) \in \mathbb{R}$

D_f $x \ne \left\{\frac{(3k+1)\pi}{2}\right\}$

Per $\frac{\pi}{\frac{2}{3}} = \frac{3\pi}{2}$

PS $\frac{\pi}{4}$ Left

Amp 1



3) $F(x) = 2\cos\left(\frac{2}{3}x + \frac{\pi}{6}\right)$

D_f $x \in \mathbb{R}$

R_f $-2 \le f(x) \le 2$

Per 3π

PS $\frac{\pi}{4}$ Left

Amp 1

Honors Trigonometry

El Quizzo

Name _____

Per 15 minutes Date _____

each of the following, state Range, Domain, Period, Phase shift (including direction) and amplitude. Graph on the supplied axis.

1) $F(x) = \sec\left(\frac{2}{3}x + \frac{\pi}{6}\right) + 1$

$\frac{2\pi}{\frac{2}{3}} = 3\pi$

D_f $x \notin \left\{\frac{(3k+1)\pi}{2}\right\}$

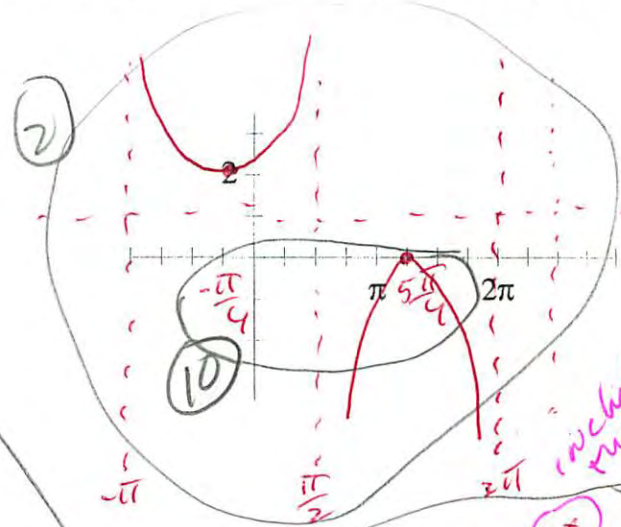
R_f $f(x) \le 0$ or $f(x) \ge 2$

Period 3π Amp 1

P.S. $\frac{\pi}{4}$ Left.

all or Nothing in this part.

(1)



(2)

(10)

(3) $\frac{2}{3}\left(x + \frac{\pi}{4}\right) = (2k+1)\frac{\pi}{2}$ Set

$x + \frac{\pi}{4} = \frac{6k\pi}{4} + \frac{3\pi}{4}$

$\frac{3k\pi}{2} + \frac{\pi}{2}$

(4) $x = \frac{(3k+1)\pi}{2}$

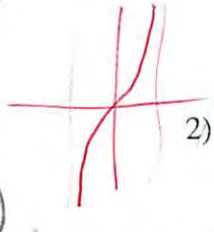
(5) $k = -1$ $x = -\pi$

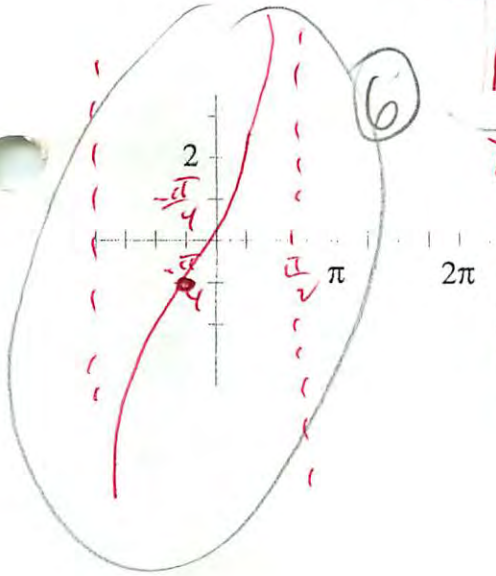
$k = 0$ $x = \frac{\pi}{2}$

$k = 1$ $x = \frac{5\pi}{2}$

including the work set

2) $F(x) = \tan\left(\frac{2}{3}x + \frac{\pi}{6}\right) - 1$





(6)

D_f $x \notin \left\{\frac{(3k+1)\pi}{2}\right\}$

R_f $f(x) \in \mathbb{R}$

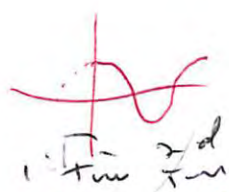
Per $\frac{3\pi}{2}$ Amp 1

P.S. $\frac{\pi}{4}$ left

(7)

must have generic pict to get credit for graph

3) $F(x) = 2\cos\left(\frac{2}{3}x + \frac{\pi}{6}\right)$



D_f $x \in \mathbb{R}$

R_f $-1 \le f(x) \le 1$

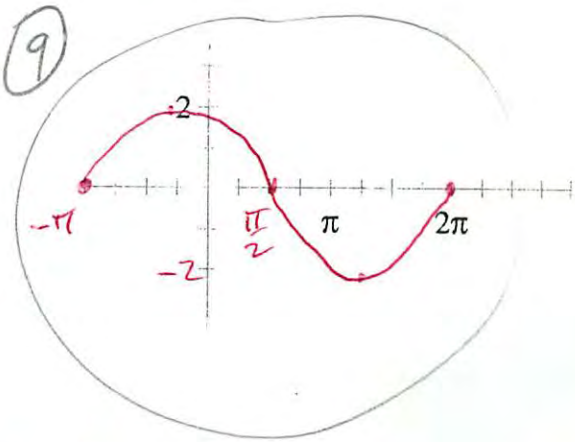
Per 3π

Amp 2

PS $\frac{\pi}{4}$ Left

(8)

1st Tim	2nd Tim	
PTS		Score
9-10	10	+
6 7 8	8,9	✓
4,5	5,6,7	0
2,3	3,4	−
0,1	0,1,2	=



(9)

circled areas 1 pt ea if Exact. "Almost" is another word for "wrong"

Honors Trigonometry

El Quizzo

Name _____

Per _____ Date _____

each of the following, state Range, Domain, Period, Phase shift (including direction) and amplitude. Graph on the supplied axis.

1) $F(x) = \csc\left(\frac{2}{3}x + \frac{\pi}{6}\right) - 1$

$\frac{2}{3}\left(x + \frac{\pi}{4}\right) = k\pi$

$x + \frac{\pi}{4} = \frac{3k\pi}{2}$

$x = \frac{(6k-1)\pi}{4}$

$k=0$ $x = -\frac{\pi}{4}$

$k=1$ $x = \frac{5\pi}{4}$

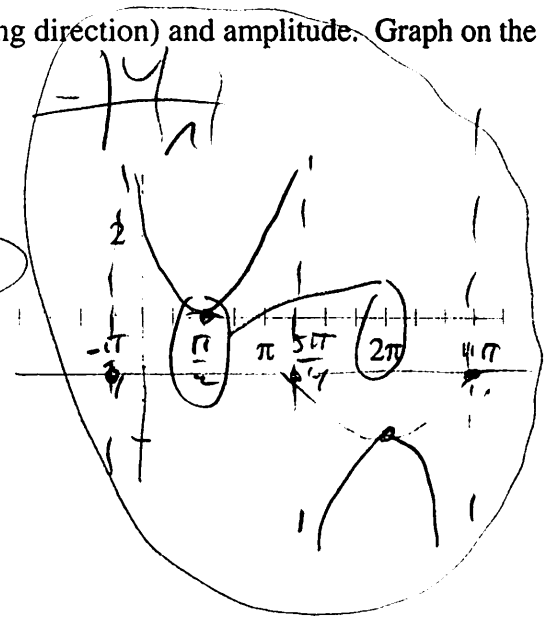
$k=2$ $\frac{11\pi}{4}$

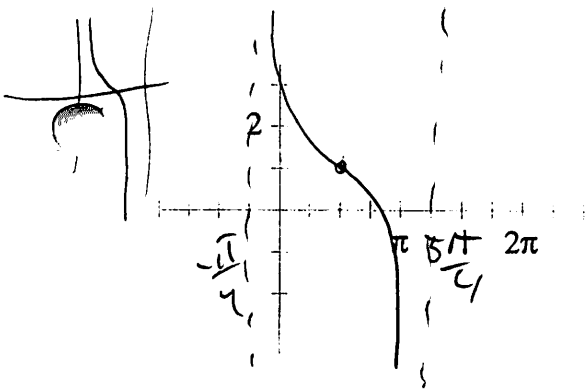
D_f $x \neq \left\{\frac{(6k-1)\pi}{4}\right\}$

R_f $f(x) \leq -2$ or $f(x) \geq 0$

Per 3π Amp 1

P.S. $\frac{\pi}{4}$ Left





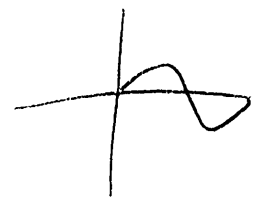
2) $F(x) = \cot\left(\frac{2}{3}x + \frac{\pi}{6}\right) + 1$

D_f $x \neq \left\{\frac{(6k-1)\pi}{4}\right\}$

R_f $f(x) \in \mathbb{R}$

Per $\frac{3\pi}{2}$ P.S. $\frac{\pi}{4}$ Left

Amp 1



3) $F(x) = 2\sin\left(\frac{2}{3}x + \frac{\pi}{6}\right)$

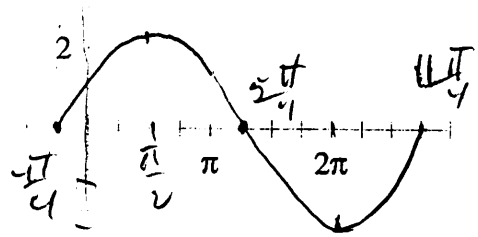
D_f $x \in \mathbb{R}$

R_f $-2 \leq f(x) \leq 2$

Per 3π

P.S. $\frac{\pi}{4}$ Left

Amp 2



Honors Trig
Chapter 3
Mr. Mumaugh

WEEKEND QUICKIE

Horatio Hypochondria has been absent from school so long that even he has forgotten where his desk is in the classroom. Horatio called ***you*** about 6:30 P.M. (Horatio, as we all know, has more brass than a band.) and begged you to "catch him up" on what he has missed in your favorite class, **TRIGONOMETRY**. Horatio seems to have forgotten everything! You start to panic and then remember that *your* awesome trig expertise will save the day for Horatio. ***You can*** change Horatio from a lowly pretender to a worthy contender! You quickly jot down the following list of topics.
You will turn in your written (and very explicit answers) to the following questions. Include drawings when they will help to clarify a point you are making but remember that pictures are not to take the place of explanations! This homework is worth 5 regular assignments. ***Only the very best and most complete responses should expect pluses.***

I A) Explain *everything there is to know* about $a, b, c,$ and d in : $\boldsymbol{a}$ **Fctn** ($\boldsymbol{bx - c}$) $+ \boldsymbol{d}$.

B) Explain how knowing which intervals on the x-axis correspond to which quadrants on the unit circle helps in drawing the graph of *any* trig function. Explain what the "argument" is. Explain how we determine the NEW period length.

C) Explain what a "generic" picture is and how we use it. Explain how the "asymptote method" and the "zeros method" are similar and how they are different and when you and why you would use each to assist in drawing the graph of a trig function.

D) Explain, in words, how to properly graph the function: $A(x) = 2\sin\left(\frac{2}{3}x - \frac{\pi}{6}\right)$

E) Draw the graph of $f(x)$. Be sure your method is VERY clear to Horatio (and me). On each picture, select 1 point on the graph which is not on an axis and estimate the (x, y) coordinate.
Compute $f($ of that x-value). Your estimate and your computed value should be very close.

1) $f(x) = 2\sin x + \cos x$ 2) $f(x) = 2\sin x \cos x$ 3) $f(x) = \sqrt{\sin^2 4x + \cos^2 4x}$

4) $f(x) = \cos\frac{1}{2}x + 2\sin\frac{3}{2}x$ 5) $f(x) = \csc^2 x - \cot^2 x$

II A) Explain what "omega" is. Explain what " rho " is.

B) Explain the difference between rotational , linear, and angular velocities.

C) Explain ***EVERYTHING*** there is to know about two points traveling around a non-unit circle at different speeds (and directions) .

D) p_1 starts at (3, 0) and p_2 starts at $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$, $\omega_1 = -\frac{\pi}{3}$ and $\omega_2 = -\frac{\pi}{4}$.

1) When will the two points coincide for the first time after $t = 0$? Use ω_1 to compute the coordinates for that first meeting. Next use ω_2 to compute the coordinates. They are the same point aren't they?

2) When AND where will the two points coincide for the **third** time after $t = 0$?

III Close your phone call from Horatio with a "Thank you." because now **you** are ready for your 3 trig exam.

← KEY →

$f(x) =$ - - - -

$g(x) =$ — • — • —

$h(x) =$ ———

I

a) $f(x) = \cos x$

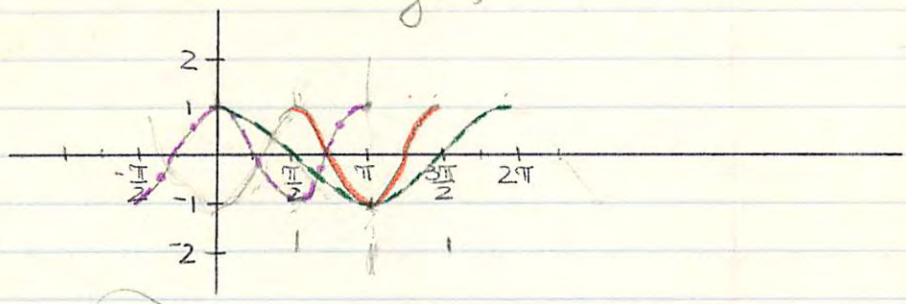
b) $g(x) = \cos 2x$

(squeeze to $x=0$)

c) $h(x) = \cos 2x$

(squeeze to $x=\pi$)

$g(x) = -h(x)$



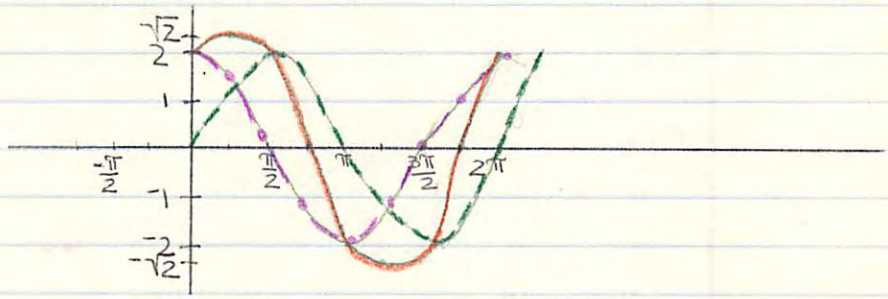
d) $\cos 2(x - \frac{\pi}{2}) = \cos 2x$

$\cos(2x - \pi) = \cos 2x \cos \pi + 0 = -\cos 2x$

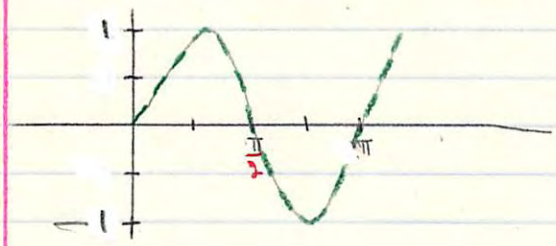
II a) $f(x) = 2\sin x$

b) $= 2\cos x$

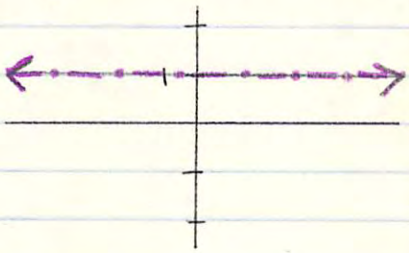
c) $= 2\sin x + 2\cos x$



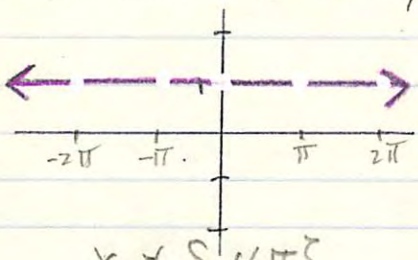
III $f(x) = 2\sin x \cos x = \sin 2x$



IV $g(x) = \sqrt{\sin^2 x + \cos^2 x}$, $g(x) > 0$



V $\csc^2 x - \cot^2 x = 1$

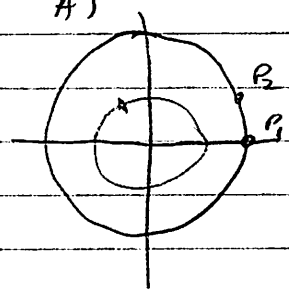


$x \notin \{k\pi\}$

Weekend Hw

VI

A) P_1 starts $(3,0)$ $\omega_1 = \frac{-\pi}{3}$; P_2 $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ $\omega_2 = -\frac{\pi}{4}$



$$\rho\omega_1 t + \frac{11\pi}{2} = \rho\omega_2 t$$

$$D_1 + \frac{11\pi}{2} = D_2$$

$$-\pi t + \frac{11\pi}{2} = -\frac{3\pi}{4} t$$

$$\frac{t}{4} = \frac{11}{2}$$

$$t = 22$$

$$\left(3\cos -\frac{\pi}{3}\cdot 22,\ 3\sin -\frac{22\pi}{3}\right)$$

$$\left(3\cos\left(\frac{4\pi}{3} + 9\cdot 2\pi\right),\ 3\sin -\frac{4\pi}{3}\right)$$

$$3\cdot\frac{-1}{2},\ 3\cdot\frac{\sqrt{3}}{2}$$

$$\left(\frac{-3}{2}, \frac{3\sqrt{3}}{2}\right)$$

Use 6π this time since neither has anyhead start,

$$D_1 - 6\pi = D_2$$

$$-t - 6 = -\frac{3t}{4}$$

$$\frac{t}{4} = 6$$

$$t = 24$$

$$3\cos\left(\frac{-24\pi}{3} + \frac{2\pi}{3}\right),\ 3\sin\left(\frac{-24\pi}{3} + \frac{2\pi}{3}\right)$$

$$3\cos\left(\frac{-22\pi}{3}\right),\ 3\sin\left(\frac{-22\pi}{3}\right)$$

$$3\cos\left(\frac{-4\pi}{3}\right),\ 3\sin\left(\frac{-4\pi}{3}\right) \Rightarrow \left(\frac{-3}{2}, \frac{3\sqrt{3}}{2}\right)$$

Same place for second coincidence.